

12/9/09

ONE Application $\rightarrow$ The Levins Model

(ref. Neuhauser)
ch. 8.2.3

Basic Idea

- observations of insect populations suggested that spatial structures (i.e. local distributions of populations) were important to maintain the overall population for a given species

- Andrewartha + Birch (1954) observed that while local populations can become extinct, their 'patches' became subsequently recolonized by migrants from other patches, thereby allowing the population to persist globally

"A natural population occupying any considerable area will be made up of a number of ... local populations or colonies."

we define these individual populations
a 'patch' (or at least, the area/region
where such a population exists)

they go on ...

"In different localities the [demographic] trends may be going in different directions at the same time."

- Continuing along in this direction, Richard Levins (American mathematical ecologist) introduced the notion of a metapopulation

Metapopulation - A collection of sub-populations. Each sub-population occupies a patch, different patches being linked via migration of individuals across patches.

- Levins developed a simple model to keep track of what proportion of patches are being occupied by a given sub-population.

Model assumptions:

- $p(t)$ is the fraction of patches occupied at time t
(clearly then $p \in [0, 1]$)

- Sub-populations go extinct at a const. ^{per capita} rate m (m -mortality) 11/2/9/09
- vacant patches can be colonized at a rate proportional to the fraction of occupied patches, the const. of proportionality being denoted by c (c -colonization)
- an increase in the fraction of occupied patches occurs only if a vacant patch becomes occupied (i.e. we can't have $p > 1$!)

⇒ putting this all together, we arrive at the following (nonlinear, first order) ODE:

Aside (questions to ask yourself)

- why is the loss term mp , not just m ?
- why is the growth term $cp(1-p)$, not just cp ?

$$\frac{dp}{dt} = cp(1-p) - mp$$

$c, m > 0$

$p \in [0, 1]$

$\underbrace{\hspace{10em}}_{\text{growth term}} \quad \underbrace{\hspace{10em}}_{\text{loss term}}$

• clearly, the metapopulation will go extinct if $p = 0$

QUESTION: For what conditions will the metapopulation persist?

⇒ start by determining if any equilibria exist:

$$\frac{dp}{dt} = 0 = cp(1-p) - mp \quad \rightarrow \quad m = c(1-p) \quad \rightarrow \quad p = 1 - \frac{m}{c}$$

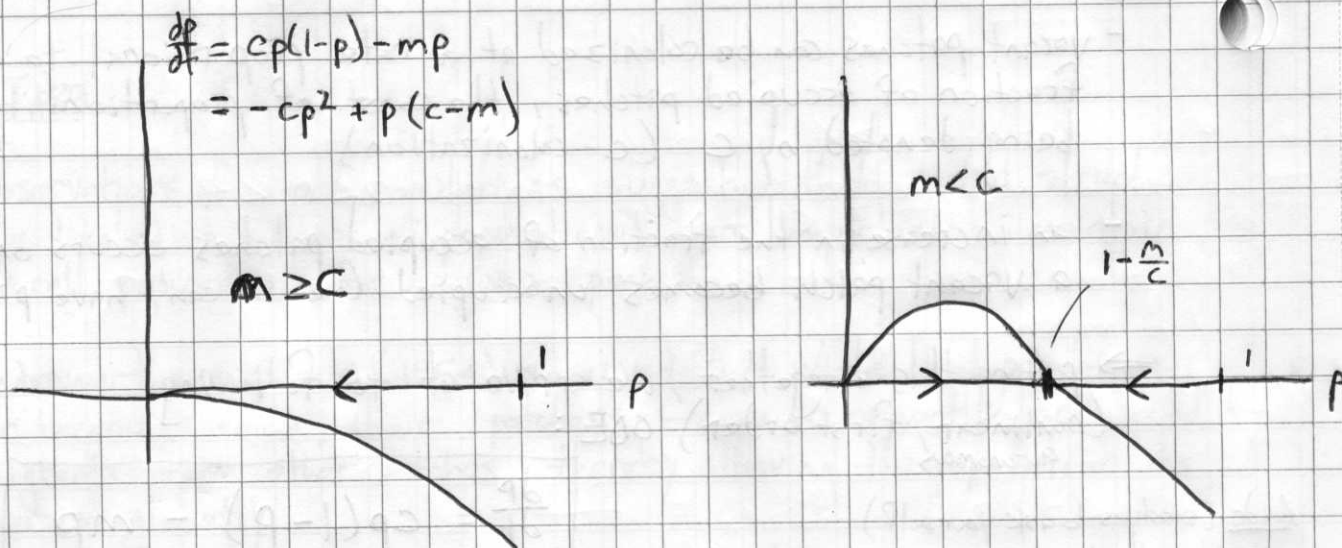
↓

$p = 0$

⇒ so we have two equilibria, $p = 0$ (extinction!) and $p = 1 - \frac{m}{c}$, the latter only existing if $1 - \frac{m}{c} > 0$ (since $p \in [0, 1]$), thus we must have ~~ANNNNN~~ $c > m$ if we are to have a non-trivial equilibrium (i.e. one where the metapopulation persists)

⇒ so we know we must have $c > m$ in order for the population to persist, but can we go further?
→ consider the phase-line diagram

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→ so when $m < c$, we see that $p = 1 - \frac{m}{c}$ is a stable equilibrium point and solution will 'tend' towards it

- applying E+U theorem, solutions exist for all values of t and p (though we only care about $p \in [0, 1]$ given the context)

- could likely solve this ODE via separation of variable, completing the square and partial fraction decomposition (which may or may not lead to a closed form for $p(t)$)

→ regardless, behavior is qualitatively similar to logistic eqn.

- might also consider 'predator-prey' meta population model: [behavior is just one possible description]

(prey) $\frac{dx}{dt} = c_x x(1-x) - m_x x (-\alpha y)$
 α, β - consts.

more predators means less prey!

(predator) $\frac{dy}{dt} = c_y y(1-y) - m_y y + (\beta x)$

→ but now we have a 'system' of coupled ODEs, which we will dig further into next semester