

□ let's come back to a recursive sequence we've already defined, the logistic map:

$$x_{n+1} = r x_n (1 - x_n) \quad [r - \text{positive const.}]$$

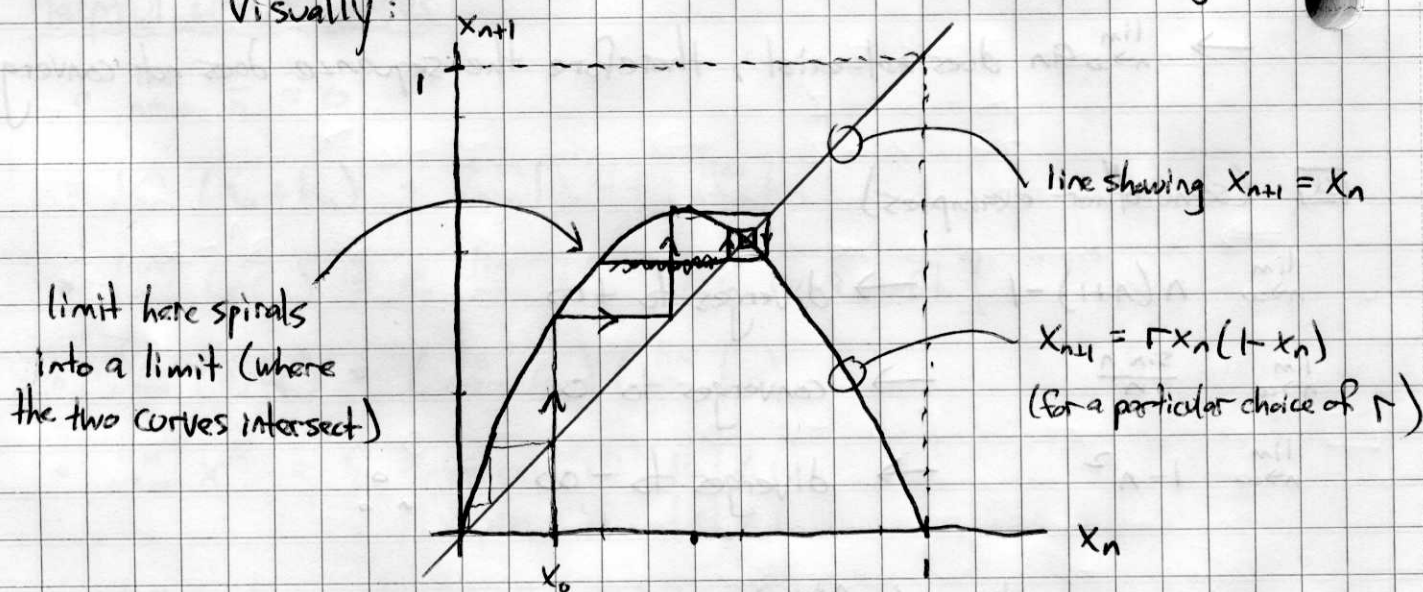
→ going to explore this 'map' in more detail. Consider a starting value x_0 . Then:

$$x_1 = r x_0 (1 - x_0)$$

$$x_2 = r x_1 (1 - x_1) = r^2 (1 - x_0) (1 - r x_0 + r x_0^2)$$

$$x_3 = r x_2 (1 - x_2) = r^3 (1 - x_0) x_0 (1 - r x_0 + r x_0^2) (1 - r^2 x_0 + r^2 x_0^2 + r^3 x_0^2 - 2r^3 x_0^3 + r^3 x_0^4)$$

→ this obviously gets complicated quickly! Clearly if $|x_0| > 1$, then the ~~same~~ sequence will diverge. If $x_0 = 1$, then you essentially have an equilibrium point. Let's simply consider the case $0 < x_0 < 1$. Then we can examine things visually:



Cobweb Plot:

• use the $x_{n+1} = x_n$ and $x_{n+1} = r x_n (1 - x_n)$ curves to determine the path subsequent iterations will take

- from x_0 , move up to $x_{n+1} = r x_n (1 - x_n)$ curve
- move left to $x_{n+1} = x_n$ curve (or to the right)
- move either up or down back to the $x_{n+1} = r x_n (1 - x_n)$ curve
- repeat

website of interest: [//math.ia.queensu.edu/~chaos/logistic.html](http://math.ia.queensu.edu/~chaos/logistic.html)

- You can change your starting value x_0 , but more importantly r as well
- Note that if $r \geq 4$, things will eventually diverge (consider why!), so we will limit our consideration to $0 < r < 4$
- different behaviors are observed for different values of r (independent of x_0):

$$0 < r < 1: \quad \lim_{n \rightarrow \infty} x_n = 0 \quad (\text{things head straight into the limit})$$

$$1 < r < 2: \quad \lim_{n \rightarrow \infty} x_n = \frac{r-1}{r} \quad (\text{things head straight into the limit})$$

$$2 < r < 3: \quad \text{ditto, but } \dots \quad (\text{will spiral in, oscillating about the limit})$$

$$r > 3: \quad \text{complex things start to happen } \dots$$

⇒ this is a very useful place to employ a bifurcation diagram