

Sequences (cont.)

• further examples...

ex find an expression for s_n for the following sequence:

0.5, 0.6667, 0.75, 0.8, 0.8333, 0.8571, ...

→ convert to fractions (these are rational #s!):

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \dots \Rightarrow \boxed{s_n = \frac{n}{n+1}}$$

(starting at $n=1$)

ex find s_n for: 1, 7, 25, 79, 241, 727

(suppose start @ $n=1$)

$n:$	1	2	3	4	5	6
$s_n:$	1	7	25	79	241	727

notice that: $7 = 9 - 2$

$$25 = 27 - 2$$

$$79 = 81 - 2 \rightarrow \text{guess } 3^n - 2?$$

⇒ plug back in to see that that works!

$$\boxed{s_n = 3^n - 2}$$

starting @ $n=1$

ex find s_n for: 1, 0, -1, 0, 1, 0, -1, 0, ...

→ alternates sign, bouncing between -1, 0 and 1 (thus not simply -1^n), suggesting something sinusoidal

~~sin~~ $\sin(\frac{\pi}{2}), \sin(0), \sin(-\frac{\pi}{2}), \sin(-\pi), \dots$
 OR $\sin(1 \cdot \frac{\pi}{2}), \sin(2 \cdot \frac{\pi}{2}), \sin(3 \cdot \frac{\pi}{2}), \dots$

$$\rightarrow \boxed{s_n = \sin(\frac{n\pi}{2})}$$

starting @ $n=1$

ex find s_n for: 0, 0.707, 1, 0.707, 0, -0.707, -1, ...

$$\rightarrow \boxed{s_n = \sin(\frac{n\pi}{4})}$$

starting @ $n=0$

12/2/09 (cont.)

- similar to a function, we can ask whether a limit exists as $n \rightarrow \infty$, or put another way, whether the sequence converges or not

Definition: A sequence $\{a_n\}$ has a limit a (where $\lim_{n \rightarrow \infty} a_n = a$) if for every $\epsilon > 0$, then there exists an integer N so that $|a_n - a| < \epsilon$ whenever $n > N$

If the limit a exists, then the series is said to be convergent.
If the sequence has no limit, it is said to be divergent.

- put another way, if a limit exists, no matter how close in you look, you can always progress further along the sequence and get even closer.

Helpful Limit 'Rules'

- $\lim_{n \rightarrow \infty} \frac{1}{n} = 0$
- $\lim_{n \rightarrow \infty} (a_n + b_n) = \lim_{n \rightarrow \infty} a_n + \lim_{n \rightarrow \infty} b_n$
- $\lim_{n \rightarrow \infty} (c a_n) = c \lim_{n \rightarrow \infty} a_n$ [c is a const.]
- $\lim_{n \rightarrow \infty} (a_n b_n) = \left(\lim_{n \rightarrow \infty} a_n \right) \left(\lim_{n \rightarrow \infty} b_n \right)$
- $\lim_{n \rightarrow \infty} x^n = \begin{cases} \text{converges if } |x| \leq 1 \\ \text{diverges if } |x| > 1 \end{cases}$
- $\lim_{n \rightarrow \infty} \frac{1}{n^p} = \begin{cases} \text{converges if } p \geq 0 \\ \text{diverges if } p < 0 \end{cases}$

ex determine $\lim_{n \rightarrow \infty} \frac{n+1}{n}$

$$\lim_{n \rightarrow \infty} \frac{n+1}{n} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right) = \lim_{n \rightarrow \infty} (1) + \lim_{n \rightarrow \infty} \left(\frac{1}{n}\right) = 1 \quad \therefore$$

ex determine $\lim_{n \rightarrow \infty} \frac{4n^2-1}{n^2}$

$$\lim_{n \rightarrow \infty} \left(\frac{4n^2-1}{n^2}\right) = \lim_{n \rightarrow \infty} \left(4 - \frac{1}{n^2}\right) = 4 - \lim_{n \rightarrow \infty} \frac{1}{n^2} = 4 \quad \therefore$$

ex exponential growth: $a_n = a_0 R^n$, find $\lim_{n \rightarrow \infty} a_n$

$$\lim_{n \rightarrow \infty} a_n = a_0 \lim_{n \rightarrow \infty} R^n = \begin{cases} 0 & R < 1 \\ a_0 & R = 1 \\ \infty & R > 1 \end{cases} \begin{matrix} \text{[converges]} \\ \text{[diverges]} \end{matrix}$$

alternates sign (see ex. below) $R = -1$

ex $a_n = \cos(\pi n)$

$n = 0$	1	2	3	4	5	...
$a_n = 1$	-1	1	-1	1	-1	...

$\rightarrow \lim_{n \rightarrow \infty} a_n$ does not exist, therefore the sequence does not converge

ex (some other examples)

$$\lim_{n \rightarrow \infty} n(n+1) - 1 \rightarrow \text{diverges to } +\infty$$

$$\lim_{n \rightarrow \infty} \frac{\sin n}{n} \rightarrow \text{converges to } 0$$

$$\lim_{n \rightarrow \infty} 1 - n^2 \rightarrow \text{diverges to } -\infty \quad \therefore$$

ex determine $\lim_{n \rightarrow \infty} \left(\frac{n+1}{n^2-1}\right)$

$$\frac{n+1}{n^2-1} = \frac{n+1}{(n+1)(n-1)} = \frac{1}{n-1} \rightarrow \lim_{n \rightarrow \infty} \left(\frac{n+1}{n^2-1}\right) = \lim_{n \rightarrow \infty} \left(\frac{1}{n-1}\right) = 0$$

could also say that for large n , $\frac{n+1}{n^2-1} \approx \frac{n}{n^2} = \frac{1}{n} \quad \therefore$