

Consider the ODE and initial condition

$$\frac{dy}{dx} = x\sqrt{y} \quad y(1) = 4$$

- a. Use Euler's method with a step-size $\Delta x = 0.1$ to estimate $y(1.1), y(1.2), y(1.3), y(1.4), y(1.5)$.

n	x_n	y_n (estimated)	exact value
0	1	4	4
1	1.1	4.2	4.21
2	1.2	4.43	4.45
3	1.3	4.68	4.72
4	1.4	4.96	5.02
5	1.5	5.27	5.34

- b. Do you think your estimated values will be exact, too big or too small? Explain.

Underestimate. Function is increasing over the interval, ^(and concave up!) so the 'left' estimate built into Euler's method will be too small

- c. Can you extrapolate as to what qualitatively you think your solution will do for larger values of x ?

Solution curves will get steeper and steeper since the rate of change keeps increasing w/ x and y . Our estimates are thus likely to get worse + worse!

- d. Is your approximated solution going to be unique? Why or why not?

Yes. The function $f(x, y) = x\sqrt{y}$ is continuous and differentiable (w/ respect to y) about $(x, y) = (1, 4)$.

- e. Determine the exact solution and compare your estimated values.

Use separation of variables, integrate and plug in the I.C. to obtain

$$y(x) = \frac{(x^2 + 7)^2}{16}$$

Group 2 (Math 250A, 11/02/09)

Consider the ODE and initial condition

$$\frac{dy}{dx} = y \quad y(0) = 1$$

a. Use Euler's method to determine $y(1)$ using 1, 2, 4, 8 and 16 steps.

N	Δx	estimated $y(1)$
1	1	2
2	0.5	2.25
4	0.25	2.44
8	0.125	2.57
16	0.0625	2.64

where $y_{n+1} = y_n + f_n \cdot \Delta x = y_n(1 + \Delta x)$
 for example, $N=2$ goes as follows:
 $y_0 = 1$
 $y_1 = (1)(1 + 0.5) = 1.5$
 $y_2 = (1.5)(1 + 0.5) = 2.25$

b. What do you think you would get if you used 32 steps? 64 steps? 43 steps? [Don't compute these by hand!]

let $E_N \equiv$ ~~accuracy~~ ^{error} of estimate using N steps (i.e. diff. between estimate and exact value)

~~$E_{32} > E_{43} > E_{64}$~~ $E_{32} > E_{43} > E_{64}$

c. Determine the exact solution. What famous # are you computing here?

$$y(x) = e^x \rightarrow y(1) = e (!)$$

$$\approx 2.7182818 \dots = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$$

d. Based upon your numerical estimates, can you infer what $y(2)$ might be? $y(3)$? $y(c)$ (where c is some # greater than 1)? How does changing the effective step-size (i.e., the number of points computed between $y(0)$ and $y(c)$) affect your estimate?

$$y(2) = e^2$$

$$y(3) = e^3$$

$$\vdots$$

$$y(c) = e^c$$

$\rightarrow$ could square our estimates above to get $y(2)$, but that would mean we are effectively ~~cutting~~ ^{doubling} our step-sizes!

Group 3 (Math 250A, 11/02/09)

The question below comes from Ch.3.3 in your ODE book (Lomen & Lovelock). In addition to the stated questions, also address the additional questions raised further below.

5. Figure 3.34 shows the slope field and a numerical solution — produced by Euler's method — for a particular initial value problem. These were generated by a computer software program. How do you know that the slope field and the numerical solution are not consistent? Which do you think is correct, the slope field or the numerical solution? What would you try to change in the software program to make them consistent?

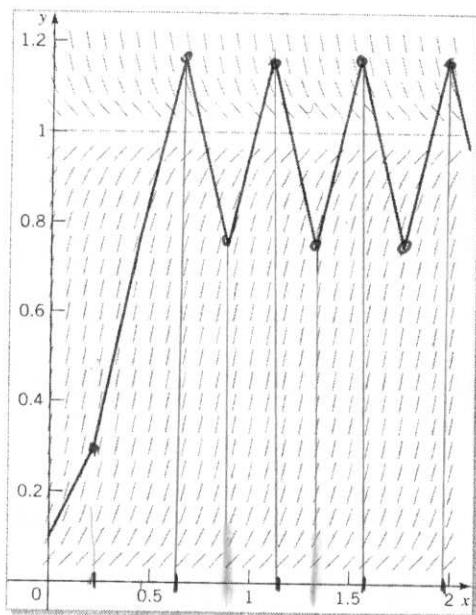


FIGURE 3.34 The slope field and a numerical solution

→ estimated solution is not consistent w/ slope field (i.e., it has a slope that is non-zero when $y=1$)

→ sharp changes in slope of solution curve (i.e. it is discontin.!!) make it suspect....

Numerical solution likely using too large a step-size, leading to an overshoot, correction, undershoot, ... (i.e. it looks like a solution is oscillating, but it's not!)

→ try using a smaller step-size (Δx)

- What was the step-size used for the numerical approximation?

Probably $\Delta x \sim 0.24$

- Can you deduce a possible ODE consistent with the figure shown?

perhaps $\frac{dy}{dx} = a x(1-x)$ where a is a positive const. likely bigger than 4

- How might you modify the ODE such that the used step-size wouldn't be so problematic?

if a was decreased, the slope wouldn't be so steep and there would be less chance of such bad overshoot as is shown above