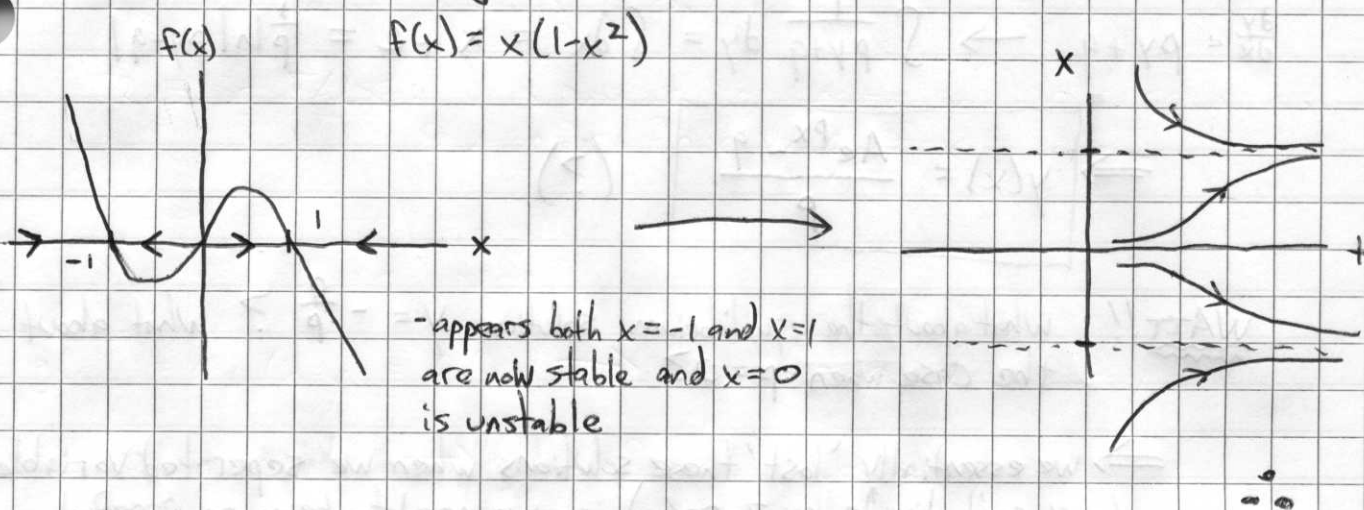


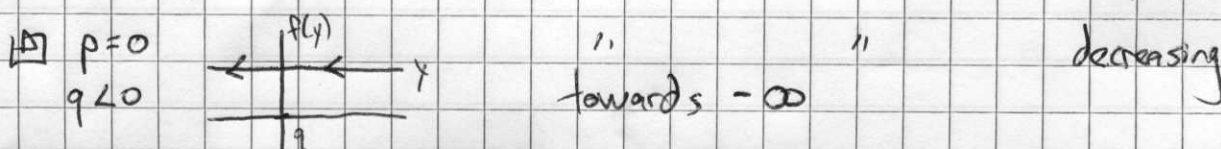
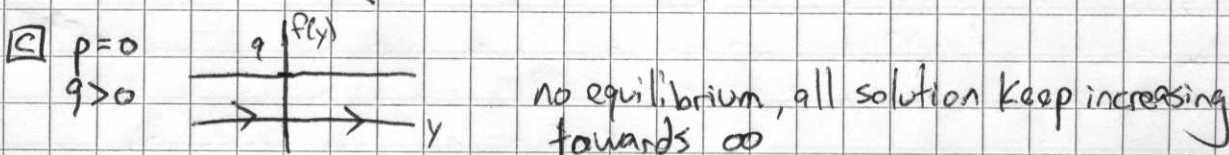
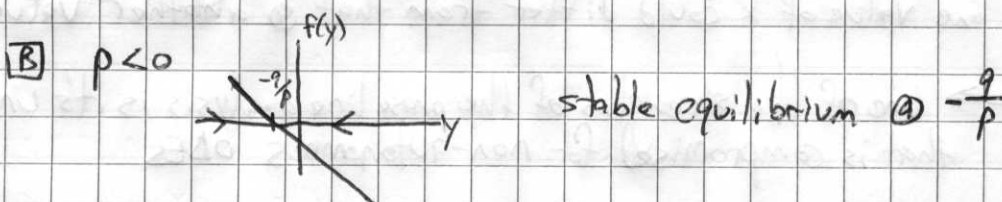
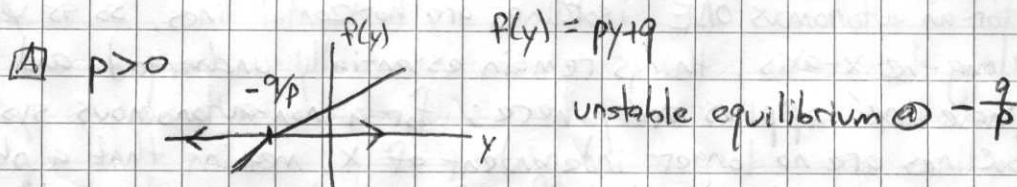
ex How do things change if we instead had $\frac{dx}{dt} = x(1-x^2)$



ex if we consider $\frac{dx}{dt} = x^a(1-x^2)$ where $a \in \mathbb{N}$, the qualitative system behavior changed as we vary a from 1 to 2; this is an example of what is called a bifurcation

ex consider the linear equation $\frac{dy}{dx} = \overbrace{py+q}^{= f(y)}$ (p, q - consts.)

→ depending upon the values of p and q , the phase line takes on different behaviors and equilibrium conditions:



* thus the phase line portrait provides a means to determine stability of equilibria

NOTE: we can also solve this ODE explicitly:

$$\frac{dy}{dx} = py + q \rightarrow \int \frac{1}{py+q} dy = \int dx = x + C = \frac{1}{p} \ln |py+q|$$

$$\Rightarrow y(x) = \frac{Ae^{px} - q}{p} \quad (?)$$

WAIT!!

What about the equilibrium solution $y = -\frac{q}{p}$? What about the case when $p=0$?

$\Rightarrow$ we essentially 'lost' those solutions when we separated variables (you can't divide by zero!), so we need to be more careful when looking for our solutions

$$\text{General solution: } y(x) = \begin{cases} 0 & y = -\frac{q}{p}, p \neq 0 \\ \frac{Ae^{px} - q}{p} & p \neq 0, y \neq -\frac{q}{p} \\ qx + C & p = 0, \end{cases} \quad A, C - \text{consts.}$$

QUESTION: Can we make a phase line portrait for a non-autonomous eqn.?
[e.g. $\frac{dy}{dx} = f(x, y)$]

$\Rightarrow$ for an autonomous ODE, isoclines are horizontal lines, so as you 'shift' along the x-axis, things remain essentially unchanged and thus the 'phase line' applies everywhere; for a non-autonomous system, isoclines are no longer independent of x, meaning that a phase line at one value of x could differ from that at another value of x

$\Rightarrow$ one of the strengths of the phase line analysis is its universality; that is compromised for non-autonomous ODEs