

ex Consider the logistic equation $\frac{dy}{dx} = ay(b-y)$. There is an equilibrium solution $y=b$.

TRUE or FALSE: It is possible for a nearby solution to reach this solution for a finite value of x .

→ use exist. + unique. theorem!

$g(x,y) = ay(b-y) = g(y) \rightarrow$ cont. for all values of y

$\frac{\partial g}{\partial y} = ab - 2ay \rightarrow$ cont. for all values of y

$\Rightarrow$ thus all solutions are unique and thus will not intersect, not even the equilibrium solution. Solutions can approach $y=b$ for $x \rightarrow \infty$, but will never touch it. **FALSE**

$\therefore$

PHASE LINE ANALYSIS (see LL 2.5)

□ a visual means to assess the qualitative behavior of autonomous ODEs, i.e. of the form $y' = F(y)$

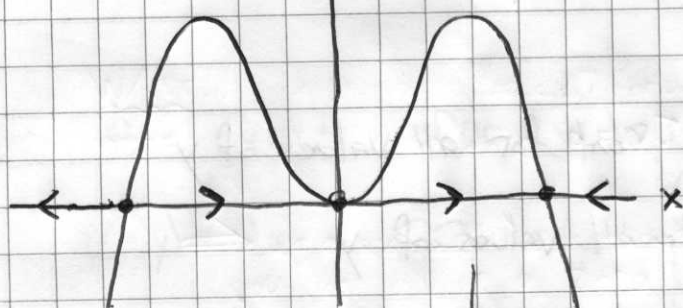
□ provides a starting point for what is called bifurcation analysis, to see how solutions to an ODE depend upon the parameters in the equation (e.g. $\frac{dx}{dt} = x^2(a-x^2)$, a is a parameter that affects the behavior of the solutions)

DEFINITION: A phase line portrait associated w/ the autonomous ODE $x' = f(x)$ consists of those subintervals of the x -axis created by the equilibria (i.e., the roots of $f(x)$) together w/ arrows pointing to the right if $f(x)$ is positive on the subinterval or to the left if $f(x)$ is negative

ex consider $\frac{dx}{dt} = x^2(1-x^2)$

$$= x^2(1-x)(1+x) \rightarrow \text{equilib. @ } x = \{-1, 0, 1\}$$

$$\frac{dx}{dt} = f(x) = x^2(1-x^2)$$



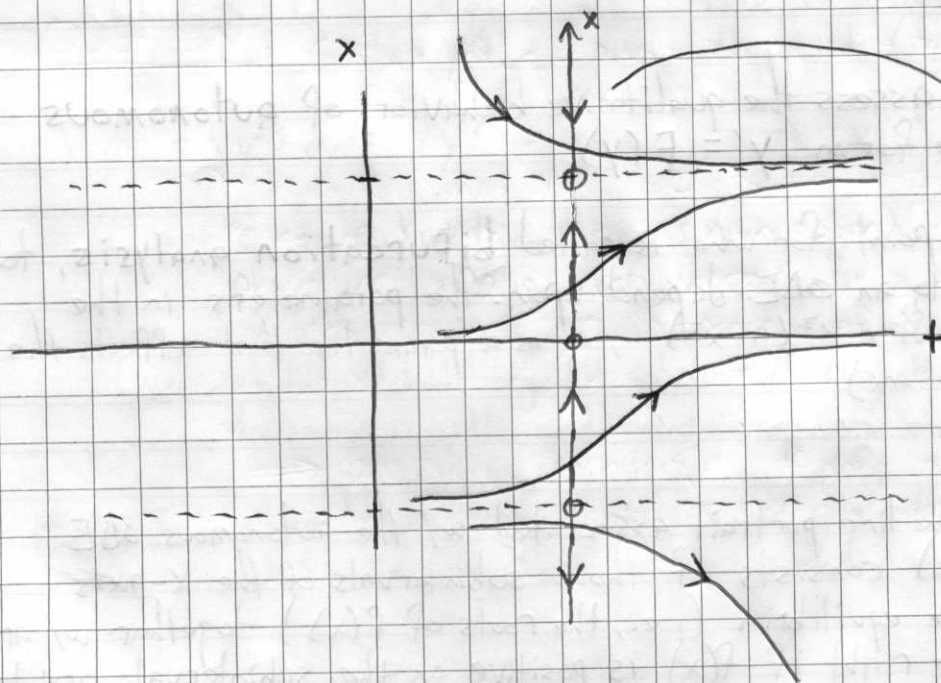
- arrows point right for $\frac{dx}{dt} > 0$
- " left for $\frac{dx}{dt} < 0$
- E+U. implies all solutions are unique (i.e. don't cross)
- Zero points represent equilibria



phase-line.



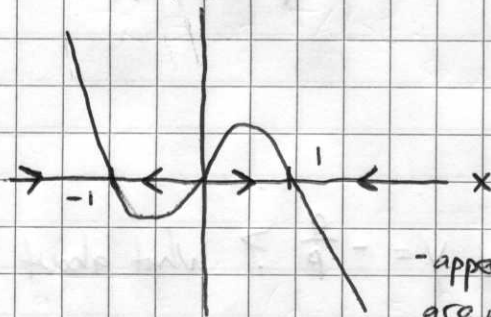
→ suggests $x=1$ is stable, $x=-1$ is unstable and $x=0$ is quasi-stable (see above diagrams); from this we can draw a picture as to what solutions will look like



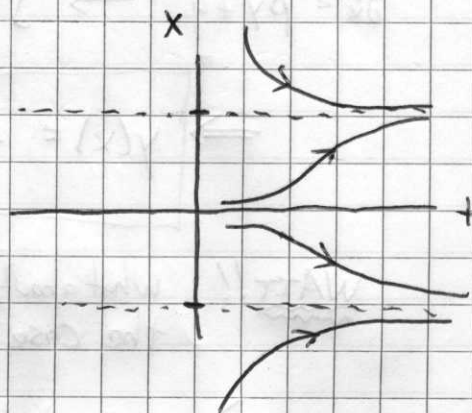
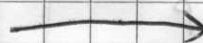
consider the phase-line as a 'portrait' along the vertical-axis of what solution curves are doing: arrows up mean a positive slope (i.e. solutions increasing) and arrows down mean solutions are decreasing

ex How do things change if we instead had $\frac{dx}{dt} = x(1-x^2)$

$f(x) = x(1-x^2)$



- appears both $x = -1$ and $x = 1$
are now stable and $x = 0$
is unstable



6

(x)

(x)

$(-x-1)x = \frac{x}{10} - 12$

