

5. Because $ay(b-y)$ satisfies the conditions of the Existence-Uniqueness theorem for all values of x and y , all solutions are unique. Therefore no other solution may intersect the equilibrium solution. Other solutions may approach the equilibrium solution as $x \rightarrow \infty$, but not touch it.

7. (a) If c is very large, many animals are being poached and the population may die out.

- (b) If $y = bY/2$, and $x = 2X/(ab)$,
 $dy/dx = d(bY/2)/dx = (b/2)dY/dx = (b/2)(dY/dX)(dX/dx) = (b/2)(dY/dX)(ab)/2$.
 This means $dy/dx = ay(b-y) - c$ becomes
 $(ab^2/4)(dY/dX) = (abY/2)[b - bY/2] - c$, or
 $dY/dX = Y(2-Y) - C$, where $C = 4c/(ab^2)$.
- (c) $dY/dX = Y(2-Y) - C$ has equilibrium solutions where $Y^2 - 2Y + C = 0$, or
 $Y = (2 \pm \sqrt{4-4C})/2 = 1 \pm \sqrt{1-C}$.
 i. If $C > 1$ there are no equilibrium solutions. ii. If $C = 1$ there is one (repeated) equilibrium solution. iii. If $C < 1$ there are two equilibrium solutions: one larger than 1, one smaller than 1.
- (d) If $C = 1$, the differential equation is $dY/dX = -(1-Y)^2$, so solutions are always decreasing and the equilibrium solution is semistable. If $C < 1$, we may write the differential equation as $dY/dX = -(Y-r_1)(Y-r_2)$, where $r_1 = 1 - \sqrt{1-C}$ and $r_2 = 1 + \sqrt{1-C}$. If $Y > r_2$, $dY/dX < 0$, while if $r_1 < Y < r_2$ we have $dY/dX > 0$. Thus $Y = r_2$ is a stable equilibrium solution. If $Y < r_1$, $dY/dX < 0$, so $Y = r_1$ is an unstable equilibrium solution.
- (e) If $C > 1$, we may write the differential equation as $dY/dX = -(1-Y)^2 - C + 1$, giving Y as a decreasing function of X . As there are no equilibrium solutions, Y will always decrease, eventually reaching a value of 0.
- (f) If $C = 1$, the population will die out if the initial condition gives $Y(0) < 1$. If $C < 1$, the population will die out if the initial condition gives $Y(0) < r_1$. (Notice it does not make sense to have $Y(0) < 0$.) The population will not die out, but have a limiting value of $1 + \sqrt{1-C}$ if $Y(0) > r_1 = 1 - \sqrt{1-C} > 0$.

9. (a) No. If the equilibrium solutions $y(x) = 1$, $y(x) = 2$, and $y(x) = 3$ were all stable, then $y(x)$ would have to be an increasing function for $y < 2$ and a decreasing function for $y > 1$. Because solutions of autonomous differential equations have slopes which do not change sign between equilibrium solutions, this is impossible.
- (b) No. The same reasoning for part (a) applies here.
- (c) Yes. One example is $y' = C(1-y)(2-y)^2(3-y)^2$, where $C > 0$. Many other examples are possible.