

Problems from Lomen + Love/ack

Ch. 2.4 (pgs. 64-65)

GROUP 2

9. Consider the differential equation $y' = g(y)$, where $g(y)$ is a given function for which dg/dy is continuous. If $y' = g(y)$ has exactly three equilibrium solutions, is it possible to write down a function $g(y)$ that satisfies the following conditions? If it is possible, write down $g(y)$ and comment on its uniqueness. If it is not possible, explain why not.

- (a) The equilibrium solutions $y(x) = 1$, $y(x) = 2$, and $y(x) = 3$ are all stable.
- (b) The equilibrium solutions $y(x) = 1$ and $y(x) = 2$ are stable, whereas $y(x) = 3$ is unstable.
- (c) The equilibrium solution $y(x) = 1$ is stable, whereas $y(x) = 2$ and $y(x) = 3$ are both semistable.

GROUP 1

5. **The Logistic Equation.** Consider the equilibrium solution $y = b$ of the logistic equation $y' = ay(b - y)$. Is it possible for a nearby solution to reach this solution for a finite value of x ? [Hint: Use the Existence-Uniqueness Theorem.]

GROUP 3

7. **Poaching.** One of the problems faced by national parks worldwide is the poaching of animals, some endangered. We will develop a very simple model to see what we can predict. We will assume that without poaching, the animal population obeys the logistic equation $y' = ay(b - y)$, where a and b are positive constants, and that poaching is done at a constant rate c , another positive constant. Thus, we assume that the animal population $y(x)$ at time x obeys the differential equation

$$y' = ay(b - y) - c. \quad (2.51)$$

(Why is there a minus sign in front of c ?)

- (a) What does your intuition suggest about the ultimate animal population if c is very large?
- (b) Show that the change of scale $y = bY/2$, $x = 2X/(ab)$ allows the differential equation (2.51) to be written as

$$\frac{dY}{dX} = Y(2 - Y) - C, \quad (2.52)$$

where $C = 4c/(ab^2)$. **Do not attempt** to solve this equation.

- (c) Find the **equilibrium solutions** of (2.52). There should be three cases.
- No equilibrium solutions.
 - One repeated equilibrium solution.
 - Two distinct equilibrium solutions.
- (d) Which equilibrium solutions are stable?
- (e) Explain why case i of part (c) predicts that the animal population will die out in a finite time. Does this agree with the answer you gave to part (a)?
- (f) Discuss under what circumstances cases ii and iii of part (c) predict that the populations will die out. What is needed for the population to survive? Under these circumstances, what is the ultimate size of the population?