

$$\square \frac{dy}{dx} = f(x) \rightarrow y(x) = \int f(x) dx$$

$$\square \frac{dy}{dx} = g(y) \rightarrow \text{separate variables + integrate} \\ \rightarrow x(y) = h(y) \hookrightarrow y(x) = r(x)$$

ex $\frac{dy}{dx} = -y$

$$\int \frac{1}{y} dy = \int -dx = -x + c = \ln|y|$$

exponentiate both sides! $y(x) = Ae^{-x}$ general solution

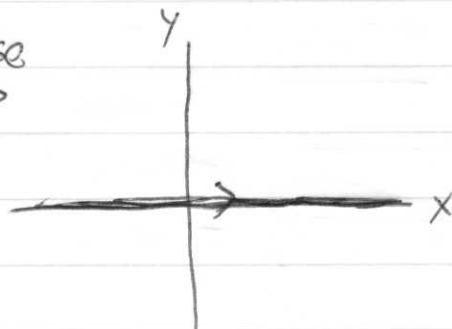
suppose we are given an 'initial condition'
 $y(0) = y_0$

$$\leadsto y(x) = y_0 e^{-x}$$
unique solution

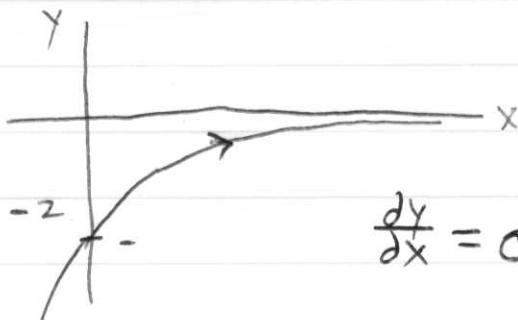
suppose
 $y_0 = 1$



suppose
 $y_0 = 0$



suppose
 $y_0 = -2$



$y = 0$ is an equilibrium solution

Stable!! (one way to think about stability is to ask what happens if I start at a point nearby)

$$\frac{dy}{dx} = 0 \rightarrow \text{defines an equil. point}$$

ex consider $\frac{dy}{dx} = y - 1$

$$\int \frac{1}{y-1} dy = \int dx$$

equilib. point: $y = 1$

$$\ln|y-1| = x + C$$

$$y-1 = Ae^x$$

suppose

I.C. $y(0) = 2$

$$y(0) = 2 = 1 + Ae^0 \rightarrow A = 1$$

$$y(x) = 1 + e^x$$

unique solution

unstable

General solution

$$y(x) = 1 + Ae^x$$

(since solutions move away from $y=1$ as x increases)

$$y(0) = 1 = 1 + Ae^0 \rightarrow A = 0 \Rightarrow y(x) = 1$$

(verifies $y=1$ is indeed an equilib. solution!)

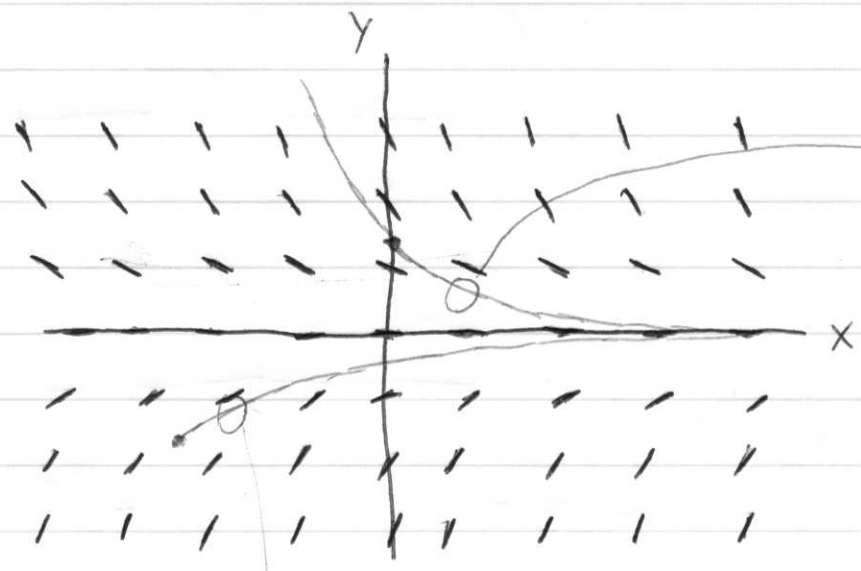
SLOPE FIELDS

$$\frac{dy}{dx} = -y$$

what does the ODE tell us about solutions?

- slopes independent of x
- further from x -axis, slopes get steeper
- if $y > 0$, slope are negative
- if $y < 0$, slope are positive

to draw the slope field



$$y(0) = 1$$

$$y(-4) = -2$$

slope field tells us what the solutions must do!!