

- 'partial fraction decomposition'

ex simplify $\frac{2x^3 - x^2 + 2x - 2}{(x^2+2)(x^2+1)}$

$$= \frac{4x+8}{x^2+2} + \frac{Cx+d}{x^2+1}$$

$$A = 2$$

$$B = 0$$

$$C = 0$$

$$D = -1$$

[see 9/30 course notes]

ex solve $\int \frac{1}{x^2-9} dx$ [factor denominator]

$$= \int \frac{1}{(x-3)(x+3)} dx$$

$$= \int \left[\frac{1}{6} \frac{1}{(x-3)} - \frac{1}{6} \frac{1}{(x+3)} \right] dx$$

$$= \frac{1}{6} \int \frac{dx}{x-3} - \frac{1}{6} \int \frac{dx}{x+3}$$

$$= \frac{1}{6} \ln|x-3| - \frac{1}{6} \ln|x+3| + C$$

$$= \frac{1}{6} \ln \frac{|x-3|}{|x+3|} + C$$

$$\frac{1}{(x-3)(x+3)} = \frac{A}{x-3} + \frac{B}{x+3}$$

$$1 = A(x+3) + B(x-3)$$

$$1 = x(A+B) + (1)(3A-3B)$$

$$x: 0 = A+B$$

$$\text{const: } 1 = 3A - 3B$$

$$A = \frac{1}{6}, B = -\frac{1}{6}$$

ex what if I asked instead: solve $\int \frac{1}{x^2+9} dx$

$$\frac{1}{x^2+9} = \frac{Ax+B}{x^2+9}$$

via table
of integrals

$$= \frac{1}{3} \arctan\left(\frac{x}{3}\right) + C$$

- various things to try:

- table of integrals
- numerical methods (e.g. your Matlab code for Riemann sums!)
- other?

good when
 $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

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ex (cont.) $\int \frac{1}{x^2+9} dx$

let $x = 3 \tan \theta$

$dx = \frac{3}{\cos^2 \theta} d\theta$

$= \int \frac{1}{9 \tan^2 \theta + 9} \frac{3}{\cos^2 \theta} d\theta$



$\tan \theta = \frac{x}{3}$

$= \frac{1}{3} \int \frac{1}{\left(\frac{\sin^2 \theta}{\cos^2 \theta} + 1\right) \cos^2 \theta} d\theta$

$\theta = \arctan\left(\frac{x}{3}\right)$

$= \frac{1}{3} \int d\theta$ (greatly simplified!)

$= \frac{1}{3} \theta + C = \frac{1}{3} \arctan\left(\frac{x}{3}\right) + C$

→ trig. substitutions