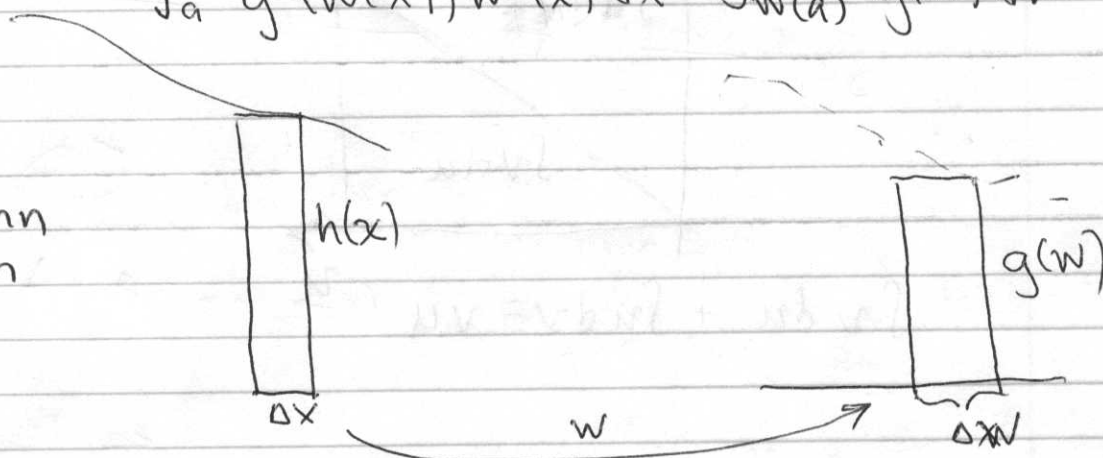


Techniques of Integration

Substitution - Chain rule

$$\int_a^b g(w(x)) w'(x) dx = \int_{w(a)}^{w(b)} g(w) dw$$

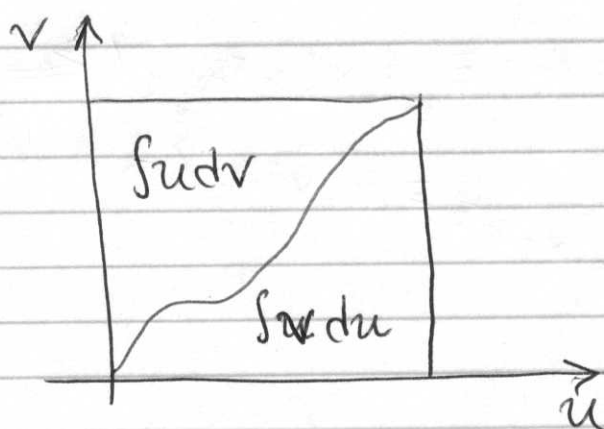
Riemann
Sum



$$h(x) \Delta x \approx g(w(x)) \Delta w$$

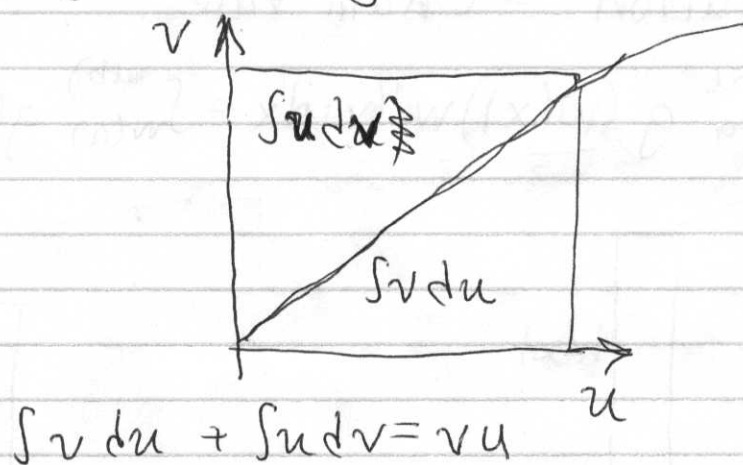
$$h(x) \approx g(w(x)) \underbrace{\frac{\Delta w}{\Delta x}}_{w'(x)}$$

Integration by parts



$$\int v du + \int u dv = uv$$

Integration by parts - product rule



②

Integration from a table

$$\int t^4 \ln(3t) dt$$

$$x = 3t$$

$$\frac{x}{3} = t$$

$$dx = 3dt$$

$$\frac{1}{3} dx = dt$$

$$\frac{1}{81} \int x^4 \ln x \frac{dx}{3}$$

$$81 = 3^4$$

$$\frac{1}{243} \int x^4 \ln x dx$$

Now use table entry 13 with $n = 4$

or

$$\int t^4 (\ln 3 + \ln t) dt$$

$$\ln 3 \int t^4 dt + \int t^4 \ln t dt$$

Using iterative formulas

$$\int \sin^5(2t+1) dt$$

$$\frac{1}{2} \int \sin^5(x) dx$$

$$x = 2t+1$$

$$\frac{dx}{2} = dt$$

Table
entry
17

$$= \frac{1}{2} \left(-\frac{1}{5} \sin^4 x \cos x + \frac{4}{5} \int \sin^3 x dx \right)$$

$$\frac{1}{3} \cos^2 x \sin x + \frac{2}{3} \int \sin x dx$$

$$-\cos x + C$$

Rational functions

- denominator is either linear or quadratic.

either the quadratic factors
or (for example)

} answer has
logarithms

$$x^2 + 4x + 5 = x^2 + 4x + 4 + 1$$

$$= (x+2)^2 + 1^2$$

$$x^2 + 6x + 11 = x^2 + 6x + 9 + 2$$

$$(x+3)^2 + (\sqrt{2})^2$$

} answer has
inverse
tangent

§ If the power in the denominator is smaller,

$$\int \frac{x}{x^2 + 5x + 4} dx = \int \frac{x}{(x+4)(x+1)} dx$$

Use table entry 27

Let's make our own table entry

5

$$\int x^n e^{-x^2} dx$$

For $n=1$

$$\int x e^{-x^2} dx$$

u-substitution

$$u = -x^2$$

$$du = -2x dx$$

$$-\frac{1}{2} \int e^{+u} du = -\frac{1}{2} e^u + C = -\frac{1}{2} e^{-x^2} + C$$

$n > 1$

$$u = x^{n-1}$$

$$v = -\frac{1}{2} e^{-x^2}$$

$$u' = (n-1)x^{n-2}$$

$$v' = x e^{-x^2}$$

$$\int x^n e^{-x^2} dx = -\frac{1}{2} x^{n-1} e^{-x^2} + \frac{(n-1)}{2} \int x^{n-2} e^{-x^2} dx$$