

Math 250A

19/23/09

P3.1

□ Prof. Watkins covering class on Friday
→ will still have quiz/HW due!

□ no office hours today; but Joseph will still have his office hours today plus recitation tomorrow as usual

'Integration by Parts' (Ch. 7.2 H.H. book)

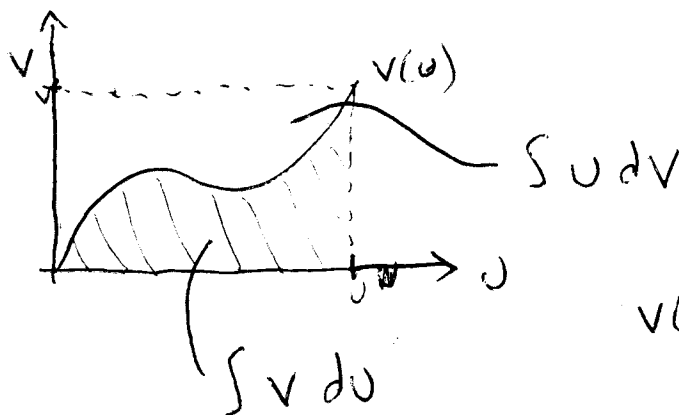
product rule: $u(x), v(x) \rightarrow \frac{d}{dx}(uv) = u'v + v'u$
[here $u' \equiv \frac{du}{dx} \dots$]

rewrite as: $(uv)' = u'v + v'u \rightarrow uv' = (uv)' - u'v$

integrate both sides (w/ respect to x):

$$\int uv' dx = \int (uv)' dx - \int u'v dx$$

$$\rightarrow \boxed{\int uv' dx = uv - \int u'v dx}$$



$$\begin{aligned} \text{total area} &= \int u dv + \int v du \\ &= uv \quad (\text{area of rectangle}) \end{aligned}$$

$$\begin{aligned} v(x) &\rightarrow dv = \frac{dv}{dx} \cdot dx \\ &= v' dx \end{aligned}$$

$$\boxed{\text{ex}} \int \overset{u}{x} \overset{v'}{\sin x} dx$$

$$u = x, u' = 1$$

$$v' = \sin x, v = -\cos x$$

pg. 2

$$= -x \cos x - \int (1)(-\cos x) dx$$

$$= -x \cos x + \int \cos x dx$$

$$= -x \cos x + \sin x + C$$

$$u = \sin x \rightarrow u' = \cos x$$

$$v' = x \rightarrow v = \frac{1}{2}x^2$$

$$\int x \sin x dx$$

$$= \frac{1}{2}x^2 \sin x - \int \frac{1}{2}x^2 \cos x dx$$

harder!

How to Choose u and v'

- 1 make sure you can figure out v from v'
- 2 it generally helps if u' is simpler than u
- 3 it generally helps if v is simpler than v'

$$\boxed{\text{ex}} \int x \ln x dx$$

$$u = \ln x \rightarrow u' = \frac{1}{x}$$

$$v' = x$$

$$v = \frac{1}{2}x^2$$

$$= \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x^2 \cdot \frac{1}{x} dx$$

$$= \frac{1}{2}x^2 \ln x - \int \frac{x}{2} dx$$

$$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$$

P9.3

ex $\int \ln x \, dx$

$$u = \ln x \rightarrow u' = \frac{1}{x}$$

$$v' = 1 \quad v = x$$

$$= x \ln x - \int x \cdot \frac{1}{x} \, dx$$

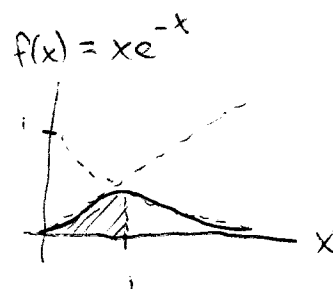
$$= x \ln x - x + C \quad \therefore$$

ex definite integral

$$\int_0^1 x e^{-x} \, dx$$

$$u = x \rightarrow u' = 1$$

$$v' = e^{-x} \rightarrow v = -e^{-x}$$



don't forget
to apply the
limits here
too!!

$$= -x e^{-x} \Big|_0^1 - \int_0^1 -e^{-x} \, dx$$

$$= -x e^{-x} \Big|_0^1 + \int_0^1 e^{-x} \, dx = -x e^{-x} \Big|_0^1 - e^{-x} \Big|_0^1$$

$$= -e^{-1} - (e^{-1} - 1) = 1 - \frac{2}{e}$$

ex $\int x^2 e^x \, dx$

$$u = x^2 \rightarrow u' = 2x$$

$$v' = e^x \rightarrow v = e^x$$

$$= x^2 e^x - 2 \int x e^x \, dx$$

$$= x^2 e^x - 2 [x e^x - \int e^x \, dx]$$

$$a = x \rightarrow a' = 1$$

$$b' = e^x \rightarrow b = e^x$$

$$= x^2 e^x - 2x e^x + 2e^x + C$$

$$= e^x [x^2 - 2x + 2] + C$$

ex $\int e^x \cos x \, dx$

$\rightarrow$ will need to use integration by parts twice, then resubstitute $\int e^x \sin x \, dx$
 $\downarrow$
 $-\int e^x \cos x \, dx$

ex $\int \cos^2 x \, dx$

~~Similar to the last case, use IBPs twice and then resubstitute~~

let $\begin{matrix} u = \cos x \\ v' = \cos x \end{matrix} \rightarrow \begin{matrix} u' = -\sin x \\ v = \sin x \end{matrix}$

$\rightarrow$ WRONG! this leads to a redundancy; use trig. identity

$$\begin{aligned} \int \cos^2 x \, dx &= \sin x \cos x + \int \sin^2 x \, dx \quad \text{use trig. identity here!} \\ &= \sin x \cos x + \int (1 - \cos^2 x) \, dx \\ &= \sin x \cos x + \int dx - \int \cos^2 x \, dx \\ &= \sin x \cos x + x - \int \cos^2 x \, dx = \int \cos^2 x \, dx \end{aligned}$$

$$\rightarrow 2 \int \cos^2 x \, dx = \sin x \cos x + x + C$$

$$\Rightarrow \int \cos^2 x \, dx = \frac{1}{2} \sin x \cos x + \frac{x}{2} + C \quad \therefore$$

similarly, one can show $\int \sin^2 x \, dx = -\frac{1}{2} \sin x \cos x + \frac{x}{2} + C$