

9/11/09

(Pg. 1)

Math 250 A

□ what is a 'differential equation'?

→ mathematical expression relating how one (or more) dependent variables change w/ respect to one (or more) independent variables

ex $\frac{d^2 y}{d\theta^2} + 9y = 0$ (i.e. $y(\theta)$)

Is $y(\theta) = \sin(4\theta) - 3\cos(4\theta)$ a solution?
 plug into ODE to find out!

$$\frac{dy}{d\theta} = y' = 4\cos(4\theta) + 12\sin(4\theta)$$

$$\frac{d^2 y}{d\theta^2} = y'' = -16\sin(4\theta) + 48\cos(4\theta)$$

$$= -16[\sin(4\theta) - 3\cos(4\theta)]$$

$$= -16y$$

$$\Rightarrow y'' + 16y = 0$$

No, not a solution
to original ODE

- can we figure out a solution?

look @ coefficients and what we did up to this point...

→ possible solution! $y(\theta) = \sin(3\theta) - 3\cos(3\theta)$

(semi-) general solution: $y(\theta) = A\sin(3\theta) + B\cos(3\theta)$
 might there be yet more solutions? (A, B - consts.)

Motivates → linear combination of solutions
 the (at present preliminary)
 notions of ... (linear independence)

□ Initial conditions → lead to a 'unique'

$$\frac{dy}{dx} = ky \rightarrow \text{general solution: } y(x) = Ae^{kx}$$

$$\text{I.C.: } y(0) = y_0 \rightarrow \text{specific unique solution: } y(x) = y_0 e^{kx}$$

again, this motivates the following notion (which we will come back to a bit later on)

⇒ Existence + Uniqueness

NOTE: I.C. need not be @ $x=0$

□ a bit more complicated: solve

$$\uparrow \frac{dn}{dt} + n = n_\infty$$

here: $n = n(t)$

$n_\infty, \uparrow$ - consts.

$$\frac{dn}{dt} = \frac{1}{\uparrow} (n_\infty - n)$$

this is very similar in nature to the example @ the top of the pg., save that there is now an additional const. on the right-hand side

solve via separation of variables and integration

$$\int \frac{1}{n_\infty - n} dn = \int \frac{1}{\uparrow} dt$$

$$-\ln |n_\infty - n| = \frac{t}{\uparrow} + C$$

$$|n_\infty - n| = e^{-t/\uparrow + C} = A e^{-t/\uparrow}$$

$$\rightarrow n(t) = n_\infty - A e^{-t/\uparrow}$$

$$\frac{dy}{dx} = ky$$



$$\frac{dy}{dx} = ay + b$$

(where $a \equiv -\frac{1}{\uparrow}, b \equiv \frac{n_\infty}{\uparrow}$)

NOTE: (hence choice of notation)

$$\lim_{t \rightarrow \infty} n(t) = n_\infty$$

→ A can be positive or negative since we drop the absolute value signs!

suppose we are given an initial condition

$$\text{I.C. } n(0) = n_0 \rightarrow n_0 = n_\infty - A e^0 \rightarrow A = n_\infty - n_0$$

$$\rightarrow n(t) = n_\infty - (n_\infty - n_0) e^{-t/\uparrow} \quad \text{unique solution}$$