

Different Types of ODEs

- general form of first order ODEs:

$$\text{or } \boxed{\frac{dy}{dx} = f(x, y)}$$

means to solve

more specifically...

$$\left. \begin{aligned} \frac{dy}{dx} &= g(x) && \rightarrow \text{readily solve via integration} \\ \frac{dy}{dx} &= h(y) && \rightarrow \text{solve via integration (autonomous ODE)} \\ \frac{dy}{dx} &= g(x)h(y) && \rightarrow \text{solve via separation of variables} \end{aligned} \right\}$$

- higher order ODEs: eg. $\frac{d^2y}{dx^2} + f(x, y) \frac{dy}{dx} + g(x, y) y = h(x, y)$

$$\frac{dny}{dx^n} \dots \rightarrow \text{break into a 'system' of } n \text{ first order ODEs}$$

- linear vs. nonlinear ODEs

DEFINITION: an ODE is said to be linear if it can be written in the form:

$$\boxed{y' + p(x)y = q(x)} \quad \text{where } y' \equiv \frac{dy}{dx}$$

$$\frac{y''}{9.5} + 1 + 3 = 0$$

$$\boxed{\text{ex}} \quad y' + y = \sqrt{1 + \cos^2 x} \quad (\text{Linear})$$

$$e^{-x} y' = \ln(2x) + (-7x^2 + 3)y \quad (\text{Linear})$$

$$y' + \cos(y)y = 3 \quad (\text{nonlinear})$$

- Autonomous ODEs: defined as ODEs of the form:

$$\boxed{\frac{dy}{dx} = g(y)}$$

$$\boxed{\text{a key example:}} \quad \frac{dy}{dx} = ky \quad \begin{array}{l} (\text{linear, 1st order}) \\ (\text{autonomous ODE}) \end{array}$$

→ looking for is $y(x)$ $k = \text{const.}$

→ exponential growth/decay

→ to solve, 'separate' and integrate

(take a look in H.H. et al. on pg. 538)

→ to put all y 'stuff' on one side, all x 'stuff' on the other and then integrate

$$\frac{1}{y} dy = k dx \quad \rightarrow \text{integrate}$$

$$\int \frac{1}{y} dy = \int k dx = kx + C = \ln|y|$$

(exponentiate both sides)

$$e^{\ln|y|} = |y| = e^{kx+C} = e^{kx} \cdot e^C = Ae^{kx}$$

$$\rightarrow \boxed{y(x) = Ae^{kx}}$$

general solution

