

8/28/09

250A Lecture

Pg. 1

LimitInformal:
definition

$$\lim_{x \rightarrow c} f(x) = L$$

 L - finite $\rightarrow$ limit converges

 L - infinite $\rightarrow$ limit diverges

one-side limits: $\lim_{x \rightarrow c^+} f(x)$ mean approaches from the right (i.e. $x > c$)
 $\lim_{x \rightarrow c^-} f(x)$ " the left (i.e. $x < c$)

ex

$$\lim_{x \rightarrow 2} x^2$$

Intuition

consider what happens when:

$$x = 1.9, 1.99, 1.999, \dots$$

$$2.1, 2.01, 2.001, \dots$$

ex

$$\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$$

rule: $x \neq 3$

$$f(x) = \frac{x^2 - 9}{x - 3} = \frac{(x-3)(x+3)}{(x-3)} = x+3$$

So we specifically
exclude $x=3$
from the domain
of f

$$\lim_{x \rightarrow 3} f(x) = \lim_{x \rightarrow 3} (x+3) = 6$$

exists

NOTE $\rightarrow f(3)$ does not existQuestion: Is $f(x)$ continuous?

(think about what the defined domain is!)

(THINK about what the graph of a function looks like)
 Definition: $f(x)$ is continuous

NOTE: assumes the function $f(x)$ is defined @ $x=x_0$

Pg. 2

When is a function continuous?

$f(x)$ is continuous @ $x=x_0$ if $\lim_{x \rightarrow x_0^+} f(x) = f(x_0)$

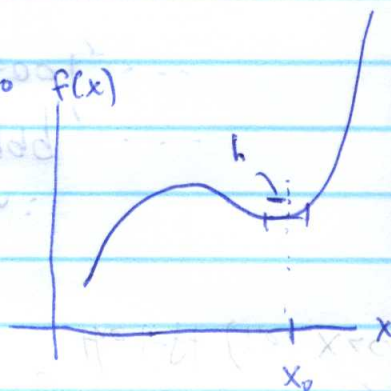
AND

$$\lim_{x \rightarrow x_0^-} f(x) = f(x_0)$$

Differentiability

$$\lim_{h \rightarrow 0} \frac{f(x_0+h) - f(x_0)}{h} = \left. \frac{df}{dx} \right|_{x=x_0} = f'(x_0)$$

(if this limit exists, then it is equal to $f'(x_0)$)



Q1 If $f(x)$ is differentiable @ $x=x_0$, is it continuous?

want to show

$$\lim_{s \rightarrow 0} f(x_0+s) = f(x_0)$$

Intuitively, it should make sense that this equals zero!!

Starting point

$$\lim_{s \rightarrow 0} |f(x_0+s) - f(x_0)| = \lim_{s \rightarrow 0} \left(\left| \frac{f(x_0+s) - f(x_0)}{s} \right| \cdot |s| \right)$$

$$= \left(\lim_{s \rightarrow 0} \left| \frac{f(x_0+s) - f(x_0)}{s} \right| \right) \cdot \lim_{s \rightarrow 0} |s|$$

$$= f'(x_0) \cdot 0 = 0$$

by definition, since we assumed $f(x)$ was differentiable @ $x=x_0$

$\Rightarrow$ If $f(x)$ is diff. @ x_0 then $f(x)$ is continuous @ x_0

Converse

$f(x) = |x| \rightarrow$ cont., but not differentiable

$\Rightarrow$ therefore, continuity does not necessarily guarantee differentiability