24-Mar-07 (draft #01)

MATH 250b (Intro to ODE’s) SYLLABUS

Fall Semester 2007: August 20– December 5

To use as a starting point, the outline below is from Math 355 (which is also a first course in ODE’s)

MAIN TOPICS

1. Modeling methodology

2. First order differential equations
    - Some basic general theory & approximation methods
    - Linear equations
    - Nonlinear equations

3. Systems of first order equations (with emphasis on 2D systems, including 2nd order equations)
    - Basic general theory & approximation methods
    - Linear systems (homogeneous & nonhomogeneous)
    - Nonlinear equations (autonomous)

SCHEDULE

Class	Topic
1	Mathematical modeling
2	Basic theory: 1st order IVP’s
3	Approximations: graphic & numerical
4	Linear equations – VC formula
5	Linear equations - shortcuts
6	Applications
7	Nonlinear autonomous equations – basic solution properties
8	Equilibria/stability/linearization
9	Phase line portraits – construction/analysis
10	Phase line portraits – analysis/classification
11	Bifurcations & bifurcation diagrams
12	Nonautonomous – separable equations (other special equations?)
13	Approximations by Taylor polynomials/perturbation methods (Picard iterates?)
14	Applications & review
15	Midterm exam #1
16	Basic theory: 1st order IVP’s for systems & higher order equations
17	Approximations: graphic & numerical, phase plane
18	Linear systems – basic theory (general solution structure, etc.)
19	Homogeneous autonomous linear systems – solution methods
20	Homogeneous autonomous linear systems – phase portraits, stability, etc.
21	Second order autonomous, higher order systems
22	Nonhomogenous linear systems – Variation of Constants formula
23	Analytic approximations by Taylor polynomials – perturbation methods
24	Applications
25	Plane Autonomous Systems (local) – Equilibria, stability & linearization
26	Plane Autonomous Systems (global) – Poincaré-Bendixson
27	Bifurcations
28	Applications & review
29	Midterm exam #2
30	Capstone topics (higher dimensions, chaos, …) & review

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MORE DETAILS

1. Basic modeling methodologies

- A "cycle" based on four steps:
  - (a) Derivation: state variable identification, symbolization, independent vs. dependent variables, parameters (coefficients) vs. state variables, rate balance, compartmental models, "laws", hypotheses/assumptions, miscellaneous jargon ("is proportional to", etc.)
  - (b) Solution: "solution" can mean an analytic formula, an analytic approximation formula, graphical or numerical approximation, approximate equation or problem
  - (c) Interpretation: use the solution to address original problem, compare to data, parameterization, model fit, predictive evaluation, errors, critique
  - (d) Modification: in light of model evaluation or changed circumstances return to derivation step.
- Several examples from various disciplines (linear, nonlinear, first order, systems, higher order)
- This format is utilized throughout the course

2. Single first order ODE's

- Basic general theory
  - Definition of solution, initial value problems, existence & uniqueness theorem
- General approximation methods
  - Slope fields
  - Numerical algorithms
    - Euler in some details, discussion of modified Euler & RK methods only
    - Order of convergence, basic checks of accuracy
- Linear equations
  - Solution formulas
    - Variation of Constants Formula (or integrating factor)
    - Shortcuts: superposition, undetermined coefficients
  - Autonomous: equilibria & phase line portrait
- Nonlinear equations
  - Autonomous
    - Equilibria & monotonicity of nonequilibrium solutions
    - Phase line portraits & their construction (geometric methods, linearization principle)
    - Stability (attractors & repellers)
    - Phase portraits analysis (bifurcations)
  - Nonautonomous
    - Separable equations
    - Taylor polynomial approximation methods (classic perturbation methods)

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3. Systems of first order ODE's (emphasis on 2D systems)

- Basic general theory
  - Definition of solution, initial value problems, existence & uniqueness theorem
- General approximation methods
  - Vector fields & numerical algorithms
- Linear systems
  - Structure of general solution
  - Homogeneous systems
    - Fundamental solution matrix
    - Autonomous systems
      - Solution methods
      - Phase portraits (nodes, saddles, spirals, etc.) & stability criteria
      - Second order equations
  - Nonhomogeneous systems
    - Variation of Constants Formula
- Nonlinear systems (autonomous)
  - Equilibria & linearization principle
  - Poincare-Bendixson Theorem (simplest form only)
  - Bifurcations of phase portraits, including Hopf theorem

Application topics (Contained in the text book)

Should we include non-biological applications (e.g. from physics)?

Most relevant (?) to biochemistry students:
- 2.6.3 (linear equations) mixing of dissolved substances
- 3.6.3 (nonlinear equations) chemical reactions – Michaelis-Menten reaction rates, growth inhibiting substrates, Hill's equation
- 4.6.1 (nonlinear systems) chemostats (Michaelis-Menten again)

Other biological applications perhaps (?) of interest to biochemistry students:
- 5.8.1 (homogeneous linear system) drug kinetics – single dose
- 6.5.1 (nonhomogeneous linear system) drug kinetics – continuous dose
- 9.7.2 (nonhomogeneous linear system) drug kinetics – periodic doses
- 1.5.1 & 9.7.2 (linear equations) - bacterial growth, control of infections

Other biological applications
- 2.6.1 (linear equations) epidemics (HIV/AIDS)
- 8.9.1 (nonlinear systems) epidemics (SIR model, HIV/AIDS)
- 3.6.1 (nonlinear equations) world population growth, self regulated populations
- 4.1 & 7.5.1 (linear systems) chemical pesticide

Other Possible Applications ?

Derive Michaelis-Menten rate equation from enzyme catalyzed reaction

$$S + K \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} C \xrightarrow{k_2} P + E$$

Math topics used: systems of ODE's, dimensionalization, perturbation (singular) approximation using Taylor polynomials, quasi-steady state hypothesis

Get some data to use for a model fit (Lineweaver-Burk & least squares)