Math 250A
Introduction to Numerical and Geometric Aspects of Functions

The first two weeks of Math 250A will be spent reviewing various ideas from Precalculus and Calculus I, but from a broader perspective. The overarching theme is a review of functions, but with an emphasis on a numerical and geometric perspective that students typically don't see in a high school course. For each type of function, we discuss the relationship between the equation of the function and the geometry of the graph of the function; the graphical meaning of each parameter; and how to deduce the parameters from some given data.

Examples and applications will be drawn from the life sciences and, where possible, biochemistry. Roughly speaking, the material breaks down as follows:

Linear functions

- Review of slope, rate of change, and derivative
- Estimating slope from data
- Least squares fit and regression
- Examples:

Exponentials and Logarithms

- Properties of exponents and logarithms, and relationships between them
- Geometry of laws of exponents: horizontal translation = vertical dilation, etc.
- How different values of a and b in $y = ae^{bx}$ affect the graph
- Graphing $\log y$ vs. x and determining a and b
- Examples: population growth, drug concentration, Newton's law of heating/cooling

Trigonometric Functions

- Properties of trigonometric functions and basic trigonometric identities
- Geometric interpretation of parameters of a trigonometric function
- Finding parameters from a data set
- Examples: circadian rhythms, predator/prey populations

Powers, Polynomials, and Rational Functions

- Comparison of power functions and exponentials
- Zeroes, asymptotes, and special values
- Finding parameters: log-log plots and reciprocal plots
- Examples: cooperative and non-cooperative ligand binding

Math 250A: Sample Lesson Plan
Rational Functions and Ligand Binding[1]

In biochemistry, a molecule that binds reversibly to a protein is called a *ligand*. The goal of this example will be to analyze protein-ligand interaction quantitatively.

Non-cooperative Binding

If the protein has a single binding site, then the *fractional saturation* θ (the fraction of binding sites that are occupied) is given by:

$$\theta = \frac{[\mathrm{L}]}{[\mathrm{L}] + K_d},$$

where [L] is the concentration of the ligand and K_d is the dissociation constant of the chemical reaction $\mathrm{P} + \mathrm{L} \leftrightharpoons \mathrm{PL}$.

1. Graph θ as a function of [L], including all zeroes and asymptotes. For what value of [L] is $\theta = \frac{1}{2}$?
2. Describe the effect on the graph as K_d varies.
3. Compute the value of $\frac{d\theta}{d[\mathrm{L}]}$ at $[\mathrm{L}] = 0$, and describe the meaning of this quantity.
4. Explain why $\frac{1}{\theta}$ is a linear function of $\frac{1}{[\mathrm{L}]}$. What are its slope and y-intercept?
5. Given the following data for the binding of insulin to an insulin receptor (in humans), estimate the dissociation constant K_d.

[L] (molar)	θ
2.43×10^{-11}	0.21
4.76×10^{-11}	0.32
7.56×10^{-11}	0.44
1.02×10^{-10}	0.51
1.27×10^{-10}	0.57
1.53×10^{-10}	0.62
1.72×10^{-10}	0.62
1.99×10^{-10}	0.67
2.27×10^{-10}	0.70
2.48×10^{-10}	0.71

Cooperative Binding

If the protein has n binding sites, then the fractional saturation θ is given by:

$$\theta = \frac{[\mathrm{L}]^n}{[\mathrm{L}]^n + K_d},$$

where [L] is, again, the concentration of the ligand and K_d is the dissociation constant of the chemical reaction $\mathrm{P} + n\mathrm{L} \leftrightharpoons \mathrm{PL}_n$.

1. Graph θ as a function of [L], including all zeroes and asymptotes. For what value of [L] is $\theta = \frac{1}{2}$?
2. Describe the effect on the graph as K_d varies.
3. Compute the value of $\frac{d\theta}{d[\mathrm{L}]}$ at $[\mathrm{L}] = 0$, and describe the meaning of this quantity.
4. $\frac{1}{\theta}$ is what kind of function of $\frac{1}{[\mathrm{L}]}$?
5. Show that $\log\left(\frac{\theta}{1-\theta}\right)$ is a linear function of $\log[\mathrm{L}]$. What are its slope and y-intercept? The graph of this linear function is called a *Hill plot*, and its slope n_H is called the *Hill coefficient*.

[1] *Lehninger Principles of Biochemistry*, Chapter 5, Section 1.