Practice for Final Exam, math 362

1. Let X be a continous random variable with probability density function

$$f_X(x) = \begin{cases} 1 - |x| & -1 < x < 1; \\ 0, & \text{otherwise} \end{cases}$$

Determine the density function for $Y = X^2$.

2. Let X and Y be continuous random variables with joint density function

$$f(x, y) = \begin{cases} 6x & 0 < x < y < 1; \\ 0, & \text{otherwise} \end{cases}$$

(A) Find $E(X)$ and $E(Y)$.
(B) Find $Cov(X, Y)$.
(C) Find $P(X < 1/2, Y > 1/2)$
(D) Determine $P(X < 1/2 \mid Y = 1)$.
(E) Determine $V[X \mid Y = 1]$

3. Suppose the number of repairs during an 8-hour workday of a machine is a Poisson random variable with mean 2.
(A) Find the probability that the machine will have 1 breakdown during the day.
(B) Find the probability that the machine will work for at least 4 hours without breaking down.
(C) Find the density function of the time between starting the machine and the second repair during an 8-hour workday.

4. An urn contains 5 black balls and 4 white balls. Suppose 4 balls are drawn from the urn without replacement. Determine the probability of obtaining 1 white ball and 3 black balls given that at least 2 of the balls drawn are black.

5. Suppose the number of claims, C, from an insurance policy is 0, 1, 2 or 3 each with probability 1/4. Let L denote the aggregate loss. Suppose that
$E(L \mid C = 0) = 0$
$V(L \mid C = 0) = 0$
$E(L \mid C = 1) = 5$
$V(L \mid C = 1) = 3$
$E(L \mid C = 2) = 10$
$V(L \mid C = 2) = 7$
$E(L \mid C = 3) = 6$
$V(L \mid C = 3) = 3$

Find $V(L)$.

6. Let the random variable X have the normal distribution with mean μ and variance σ^2. Find the density function of $Y = e^X$.

7. Suppose a test for a disease is positive with probability 0.90 if a person has the disease. The test is negative with probability 0.95 if the person does not have the disease. If 1% of the population has the disease, determine the probability that a person has the disease given that he/she tests positive for the disease.

8. Suppose a six sided die is rolled until two "ones" are observed. Let X be the number of times the die is rolled before two "ones" are observed.
(A) What is the probability that two "ones" are observed by rolling the die no more than 10 times?
(B) Find the expected value and variance of the number of times the die is rolled until two "ones" are observed.

9. Suppose that a survey shows that all members of a population either owns a house or have a driver's license, and that those two events are independent of each other. Suppose that for a person randomly selected from the population, there is a probability of p $(0 \leq p < 1)$ that the person has a driver's license. Determine the probability that a randomly selected person from the population owns a house.

10. Suppose that a company buy lightbulbs from three suppliers-I, II, and III. A record shows that 5% of the bulbs from supplier I are defective, 3% of the bulbs from supplier II are defective, and 1% of the bulbs from supplier III are defective. Suppose 10%, 30% and 60% of the current supply came from suppliers I, II, and III, respectively.
(A) If a light bulb is randomly selected from this supply, what is the probability that it is defective.
(B) If a randomly selective bulb is defective what is the probability that it came from supplier III.

11. (Exercise 5.97)
A machine for filling cereal boxes has a standard deviation of 1 ounce in fill per box. Assume that the ounces of fill per box are normally distributed.
(A) What setting of the mean ounces of fill per box will allow 14-ounce boxes to overflow only 1% of the time?
(B) The company advertises that the box holds 12.8 ounces of cereal. Using the mean determined in part (a), what is the probability that a randomly selected box will have less than 12.8 ounces of cereal?

12 Suppose 8 people are testing a new food product and the company will go forward with the new product if at least 6 of the people prefer the new product to the old one. Suppose 20% of all people cannot tell the difference between the two and so will randomly choose one of them. Of the remaining people, 80% prefer the new product.
What is the probability that the company will move forward with the new product.

13. Suppose that two friends plan to meet in the park during a given 1-hour period. Their arrival are independent and randomly distributed across the 1-hour period. They agree to wait for each other for 15 minutes or until the end of the hour or they will leave. What is the probability that they meet?