Practice for Exam 3, math 362

1. An insurance company divides its policyholders ino good or bad drivers. Suppose that 20% of its policyholders are considered bad drivers. The insurance company estimates that for a bad driver, the probability of n claims in a year is given by a Poisson distribution with mean 0.5. For a good driver, the probability of n claims in a year is given by a Poisson distribution with mean of 0.1. Determine the expected number of claims this year for a policyholder who had one claim on his insurance policy last year.

2. Let X be a continous random variable with probability density function

$$f_X(x) = \begin{cases} 2x & 0 < x < 1; \\ 0, & \text{otherwise} \end{cases}$$

Let Y be a continous random variable with a conditional distribution of Y given $X = x$, $f_{Y|X}(y \mid x)$, which is uniform on $(0, x)$. Determine the mean and variance of Y.

3. Let X and Y be continuous random variables with joint density function

$$f(x, y) = \begin{cases} 8xy & 0 < x < 2,\ 0 < y < 2 - x; \\ 0, & \text{otherwise} \end{cases}$$

(A) Find $P(X > 1, Y < 1/2)$.
(B) Find the marginal density function for X.
(C) Determine $P(Y < 1/2 \mid X = 1)$.
(D) Determine $V[Y \mid X = 1]$

4. Suppose that X has a normal distribution with mean 3 and variance 16. Determine $P(X^2 - 6X < -5)$.

5. Suppose that the number of phone calls passing trough a particular cellular relay follows a Poisson distribution with an average of 4 calls during a 2-min period.
(A) Find the probability that no call will pass through the relay system during a given 2-min period.
(B) Find the probability that at least 3 minutes will pass before a call is passing through the relay system.

6. Suppose that three certain tasks have to be completed. Completion of the task has an exponentially distributed time with mean of 5 hours. The time it takes to complete a task is independent of the completion time of the other ones.
(A) Give the probability density function, including parameters, of the total time to complete all three tasks.
(B) Determine the expected value and the variance of the total time to complete all three tasks.

7. Let X and Y be continuous random variables with joint density function

$$f(x, y) = \begin{cases} xy & 0 < x < 2,\ 0 < y < 1; \\ 0, & \text{otherwise} \end{cases}$$

Define $Z = X + Y$
Find $V(Z)$.