

Review for exam 2

Show all your work and use proper notation. Define all the events that you are using.

1. In adults the pneumococcus bacterium causes 70% of pneumonia cases. In a random sample of $n = 84$ adults who have pneumonia, use the Normal distribution to approximate the probability that at most 52 samples of pneumonia was caused by the pneumococcus bacterium.

Answer: 0.077

2. Suppose the time Linda spends commuting to and from work each day is on average 1.1 hour with a standard deviation of 0.2 hour.

(A) Find the probability that the **average** daily commute time for a period of 25 working days is between 1 and 1.2 hours.

(B) Find the probability that the total **commute** time for the 25 days is less than 28 hours.

Answer:

(A) 0.988

(B) 0.691

3. Customers at a football game win a 50 dollar prize if the ticket receipts show a star on each of the three consecutive days Friday, Saturday and Sunday in any one week. The probability of receiving a star is 0.2 on each of the tickets and the appearance of a star is an independent process. If a customer goes to each game for five consecutive weeks what is the expected number of dollars won by the customer in the five-week period?

What is the variance of the dollar won by the customer in the five-week period?

Answer: Expected number of dollars won: $50 \cdot 5 \cdot 0.008$ dollar = 2 dollar. The variance of the dollar won: $(50)^2 \cdot 5 \cdot 0.008 \cdot 0.992$ dollar² = 99.2 dollar².

4. The life time, X , of a certain electrical component follows an exponential distribution with a mean of 200 hours.

Find the probability that an electronic component last between 100 and 300 hours.

Answer: $-e^{-\frac{3}{2}} + e^{-\frac{1}{2}}$

5. A machine for filling cereal boxes has a mean of 12 ounces of fill per box with a standard deviation of 1.2 ounces in fill per box. Assume the ounces of fill per box are normally distributed.

(A) Suppose a cereal box is randomly selected. What is the probability that the box holds more than 12.5 ounces of cereal.

(B) Suppose a sample of 9 cereal boxes are randomly selected. What is probability that at least 8 of those boxes holds more than 12.5 ounces of cereal.

(C) Suppose a sample of 9 cereal boxes are randomly selected. What is the probability that the sample mean of fill per box of those 9 samples is greater than 12.5 ounces?

Answer:

(A) 0.338

- (B) 0.00107
- (C) 0.106

6. In a study of maximal aerobic capacity, 12 women were used as subjects, and one measurement that was made was blood plasma volume. They obtained that the sample mean of their blood plasma volumes in liters was $\bar{x} = 3.24$ and the standard deviation was $s = 0.49$. Assume that these are observations of a normally distributed random variable X that has mean μ and standard deviation σ .

Give a 98% confidence interval for μ .

Answer: (2.86, 3.62)

7. Let the random variable X have p.d.f $f(x) = \frac{1}{2\sqrt{x}}$ for $0 < x < 1$.

- (A) Find the mean and variance of X .
- (B) Find the third quartile.
- (C) Find $P(0 < X < 0.5)$.

Answer:

- (A) $E(X) = \frac{1}{3}$ and $Var(X) = \frac{4}{45}$
- (B) $\frac{\sqrt{3}}{2}$
- (C) $\sqrt{0.5}$

8. Let the independent random variables X_1 and X_2 have parameters $\mu_1 = 5$, $\mu_2 = 1$, $\sigma_1 = 16$ and $\sigma_2 = 4$, respectively. Find the mean and variance of $Y = 4X_1 - 3X_2 + 2$.

Answer:

$E(Y) = 19$ and $Var(Y) = 220$

9. Let $f(x; \theta) = \theta x^{\theta-1}$ for $0 < x < 1$ and $\theta \in \Omega = \{\theta : 0 < \theta < \infty\}$. Let X_1, X_2, \dots, X_n denote a random sample of size n from this distribution. Find the maximum likelihood estimator of θ .

Answer: $\hat{\theta} = -\frac{n}{\ln(\prod_{i=1}^n X_i)}$