

## 1. REVIEW FOR EXAM 2

1. (HGM calc 2.4.1) The temperature,  $H$ , in degrees Celsius, of a cup of coffee placed on the kitchen counter is given by  $H = f(t)$ , where  $t$  is minutes since the coffee was put on the counter.

(A) What does  $f(20) = 90$  mean? (give units)

(B) What does  $f'(20) = -10$  mean? (give units)

2. Is the function

$$f(x) = x|x|$$

differentiable everywhere? (show your work)

3. (HGM calc 2.6.13) A magnetic field,  $B$ , is given as a function of the distance,  $r$  from the center of a wire as follows:

$$B(r) = \begin{cases} \frac{r}{r_0} B_0 & \text{for } r \leq r_0 \\ \frac{r_0}{r} B_0 & \text{for } r > r_0. \end{cases}$$

(A) Is  $B$  continuous at  $r = r_0$ ? (Show your work).

(B) Is  $B$  differentiable at  $r = r_0$ ? (Show your work)

(C) Sketch a graph of  $B$  against  $r$ .

4. Given the function

$$g(t) = \begin{cases} 2 + \sin(2\pi e^t) & \text{for } t \leq 0 \\ 2(3^t) & \text{for } t > 0. \end{cases}$$

(A) Is the function  $g$  continuous in  $t = 0$ ?

(B) Is the function  $g$  differentiable in  $t = 0$ ?

5. (HGM calc 3.3.53) The quantity,  $q$ , of a certain skateboard sold depends on the selling price,  $p$ , in dollars, so we write  $q = f(p)$ . You are given that

$$f(140) = 15000$$

and

$$f'(140) = -100.$$

(A) What do  $f(140) = 15000$  and  $f'(140) = -100$  tell you about the sales of skateboards?

(B) The total revenue,  $R$  earned by the sale of skateboards is given by

$$R = pq.$$

Find

$$\left. \frac{dR}{dp} \right|_{p=140}$$

(C) What is the sign of  $\left. \frac{dR}{dp} \right|_{p=140}$ ? If the skateboards are currently selling for 140, what happens to revenue if the price is increased to 141?

6. For what interval(s) is

$$h(x) = x2^{bx}, \quad \text{where } b > 1 \text{ is a constant,}$$

concave down?

7. Find the equation of the tangent line to

$$f(x) = (x - 2)^4$$

at  $x = 3$ .

8. Give an example of a function  $f$  with negative, but everywhere increasing slope. What is the sign of  $f''$ ?

9. If

$$\frac{d}{dt}(tf(t)) = 2 + f(t),$$

what is  $f'(t)$ ?

10. Suppose  $f(x)$  is increasing and concave up everywhere and  $f(A) = 4$ ,  $f'(A) = 2.2$ ,  $h = 0.05$ . Estimate the values of  $f(A - h)$  and  $f(A + h)$ . Hint: Use the definition of the derivative.

11. Given the table

x	f(x)	f'(x)	g(x)	g'(x)
1	2	-1	3	4
2	3	5	2	1

Assume that the function  $f$  is invertible and differentiable everywhere.

A Find  $(f^{-1})'(2)$ .

B Determine  $h'(2)$  if  $h(x) = \frac{\sqrt{f(x)}}{x}$ .

C Determine  $k'(1)$  if  $k(x) = f(x)g(x)$ .

D Determine if the function in part C is increasing or decreasing at  $x = 1$ .