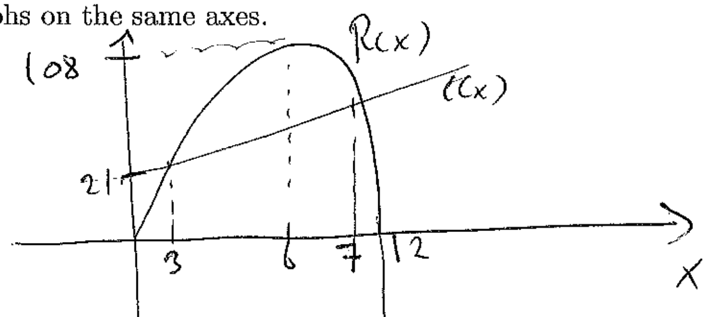
Review for Exam 1

1. Let $C(x) = 2x + 21$ be the cost to produce x batches of widgets and let $R(x) = -x^2 + 12x$ be the revenue in thousands of dollars.

$R(x) = -x(x-12)$

(A) Draw the graphs on the same axes.



(B) Find the maximum revenue

$-\frac{b}{2a} = \frac{-12}{2(-1)} = 6$

Max revenue when the quantity is 6 batches of widgets.

Max revenue is $R(6) = -6^2 + 12 \cdot 6 = 108$

Max revenue is $\underline{\$ 108}$

(C) Find the minimum break-even quantity.

$C(x) = R(x)$ $\quad 2x + 21 = -x^2 + 12x$

$-x^2 + 10x - 21 = 0$

$(-x+7)(x-3) = 0$

$x = 3, \quad x = 7$

Minimum break-even quantity is 3 batches of widgets.

(D) Find the profit from selling 6 batches of widgets.

$P(x) = R(x) - C(x)$

$= -x^2 + 12x - (2x+21)$

$= -x^2 + 10x - 21$

$P(6) = -6^2 + 10 \cdot 6 - 21 = 75$

The profit is $\underline{\underline{\$ 75}}$

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2. The revenue in thousands of dollars from producing x units of an item is

$$R(x) = 10x - 0.05x^2.$$

(A) Find the average rate of change of revenue when production is increased from 101 to 102 units.

$$\frac{R(102)-R(101)}{102-101} = \frac{499.8 - 499.95}{1} = -0.15$$

$R(102) = 10\cdot(102) - 0.05\,(102)^2 = 499.8$

$R(101) = 10\cdot(101) - 0.05\,(101)^2 = 499.95$

The average rate of change when production is increased from 101 to 102 is -150 dollar per unit.

(B) Use the definition of the derivative as the limit of the difference quotient as $h \to 0$ to find the marginal revenue at $x = 101$. Interpret the result.

$$R'(101) = \lim_{h\to 0} \frac{R(101+h) - R(101)}{h} = \lim_{h\to 0} \frac{10(101+h) - 0.05(101+h)^2 - (10\cdot 101 - 0.05(101)^2)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{1010} + 10h - \cancel{0.05}((\cancel{101})^2 + 2\cdot 101\cdot h + h^2) - \cancel{1010} + \cancel{0.05(101)^2}}{h}$$

$$= \lim_{h\to 0} \frac{10h - 10.1h - 0.05h^2}{h} = \lim_{h\to 0} \frac{(-0.1h - 0.05h^2)}{h}$$

$$= \lim_{h\to 0} -0.1 - 0.05h = -0.1 - 0.05\cdot 0 = -0.1$$

The marginal revenue is -100 dollar

Thus when 101 units are produced the revenue decreases by \$100 per unit

That is, when 101 units are produced, the revenue will decrease by \$100 for an additional unit produced.

(C) Find the additional revenue if production is increased from 101 to 102 units.

$$R(102) - R(101) = 499.8 - 499.95 = -0.15$$

The additional revenue is -150 dollar.

3. Given the function

$$g(x) = \frac{2x^2 - 4}{x^2 - 9}$$

(A) Find any asymptotes of $g(x)$.

Vertical asymptotes: $x^2 - 9 = 0$

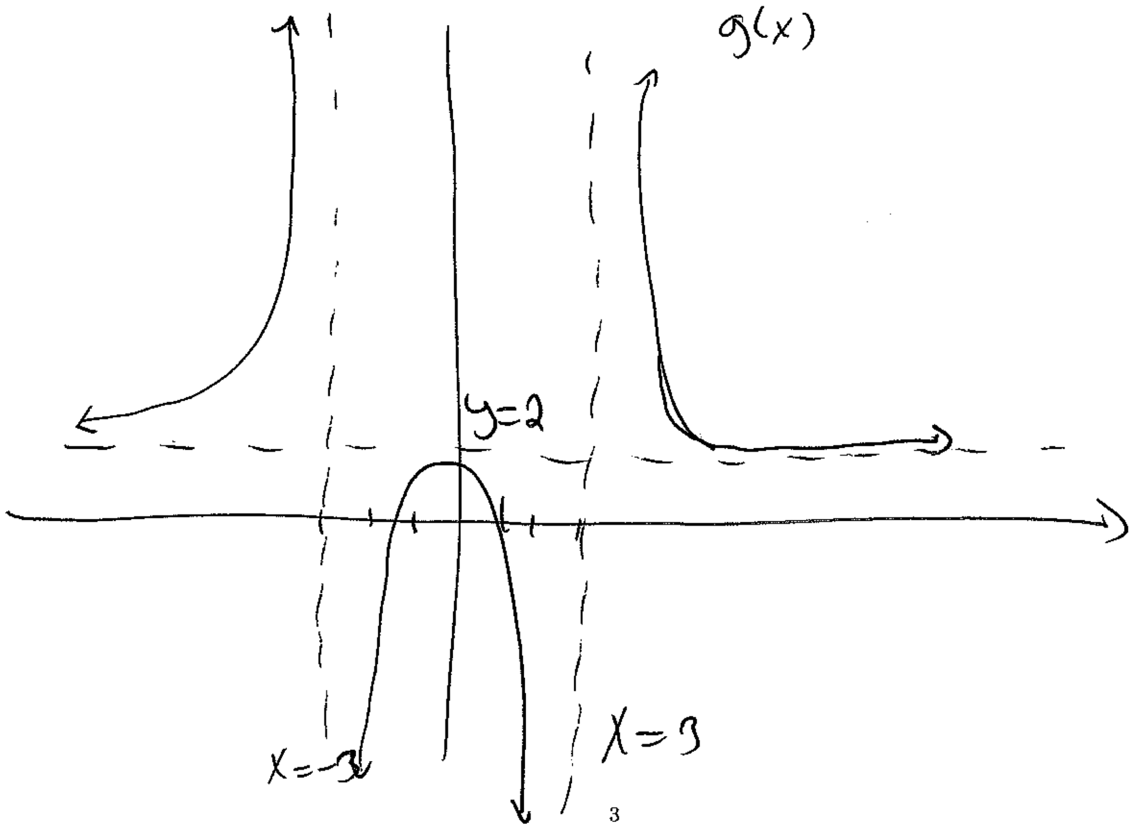
$$\boxed{x = \pm 3}$$

Horizontal asymptotes: $\lim\limits_{x \to \infty} \frac{2x^2-4}{x^2-9} = \lim\limits_{x \to \infty} \frac{\frac{2x^2}{x^2} - \frac{4}{x^2}}{\frac{x^2}{x^2} - \frac{9}{x^2}}$

$$= \lim_{x \to \infty} \frac{2 - \frac{4}{x^2}}{1 - \frac{9}{x^2}} = \frac{2-0}{1-0} = 2$$

$$\boxed{y = 2}$$

(B) Graph the function $g(x)$. Include any asymptotes.



4. Eva wants to invest \$8000 in a fund. Fund A offers 5.3% compounded monthly. Fund B offers 5.0% compounded continuosly.

(A) How much money will be in the fund after 10 years if she chooses fund A

$$A = 8000\left(1 + \frac{0.053}{12}\right)^{10\cdot 12} = \$13575.61$$

(B) How much money will be in the fund after 10 years if she chooses fund B

$$A = 8000\, e^{0.05\cdot 10} = \underline{\underline{\$13189.77}}$$

(C) Calculate the effective rate in each case.

Fund A: $r_E = \left(1 + \frac{0.053}{12}\right)^{12} - 1 = \underline{0.0543}$

Effective rate for fund A is $\underline{\underline{5.43\%}}$

Fund B: $r_E = e^{0.05} - 1 = 0.0513$

Effective rate for fund B is $\underline{\underline{5.13\%}}$

5. (A) If \$2000 is deposited in an account and the amount of money after 7 years is \$2700 compounded continuously. What is the interest rate?

$$2700 = 2000 \cdot e^{r \cdot 7}$$

$$\frac{2700}{2000} = e^{7r}$$

$$\ln\left(\frac{27}{20}\right) = 7r$$

$$r = \ln\left(\frac{27}{20}\right) \cdot \frac{1}{7} = 0.0429$$

Interest rate is 4.29 %

(B) Luis wants to have \$30,000 in 5 years for a down payment on a new house. How much should he deposit today at 4% compounded quartely, to have that amount in 5 years?

$$30\,000 = P\left(1 + \frac{0.04}{4}\right)^{5 \cdot 4}$$

$$30\,000 = P\,1.220$$

$$P = \frac{30000}{1.220} = 24\,590.16$$

Principal value is \$ 24590.16

6. Consider the function

$$f(x) = \begin{cases} x^2 - 2 & \text{if } x \leq 5; \\ 2x + 3 & \text{if } x > 5. \end{cases}$$

Determine if the limit $\lim_{x \to 5} f(x)$ exists

$$\lim_{x \to 5^-} f(x) = \lim_{x \to 5^-} x^2 - 2 = 5^2 - 2 = 23$$

$$\lim_{x \to 5^+} f(x) = \lim_{x \to 5^+} 2x + 3 = 2 \cdot 5 + 3 = 13$$

So $\lim_{x \to 5} f(x)$ does not exists.

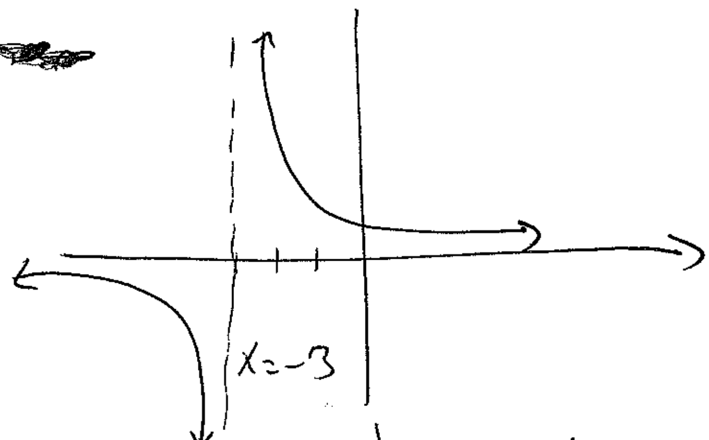
7. Find the limits if the limits exists:

(A) $\lim\limits_{x\to 4} \dfrac{\sqrt{x}-2}{x^2-x-12} = \lim\limits_{x\to 4} \dfrac{\sqrt{x}-2}{(x+3)(x-4)} = \lim\limits_{x\to 4} \dfrac{(\sqrt{x}-2)(\sqrt{x}+2)}{(\sqrt{x}+2)(x+3)(x-4)}$

$= \lim\limits_{x\to 4} \dfrac{\cancel{x-4}}{(x+3)\cancel{(x-4)}(\sqrt{x}+2)} = \lim\limits_{x\to 4} \dfrac{1}{(x+3)(\sqrt{x}+2)} = \dfrac{1}{(4+3)(\sqrt{4}+2)}$

$= \dfrac{1}{7\cdot 4} = \underline{\underline{\dfrac{1}{28}}}$

(B) $\lim\limits_{x\to -3} \dfrac{1}{x+3}$ does not exists



(C) $\lim\limits_{x\to 7} \dfrac{3x(x-7)}{x^2-49} = \lim\limits_{x\to 7} \dfrac{3x\cancel{(x-7)}}{(x+7)\cancel{(x-7)}} = \lim\limits_{x\to 7} \dfrac{3x}{x+7}$

$= \dfrac{3\cdot 7}{7+7} = \underline{\underline{\dfrac{21}{14}}}$

8. Joanne sells silk-screened T-shirts at community festivals and crafts fairs. Her marginal cost to produce one T-shirt is $3.50. Her total cost to produce 60 T-shirts is $300, and she sells them for 9 each.

(A) Find the linear cost function for Joanne's T-shirt production.

Cost function: $C(X) = mX + b$

$C(x) = 3.50X + b$

(60, 300) is a point on the graph: Since $C(60) = 300$

$y - 300 = 3.5(X - 60)$

$y = 3.5X - 210 + 300 \Rightarrow y = 3.5X + 90$

$C(x) = 3.5X + 90$

(B) How many T-shirts must she produce and sell in order to break even?

Revenue: $R(x) = 9 \cdot X$

$R(x) = C(x)$

$9X = 3.5X + 90$

$5.5X = 90$

$X = \frac{90}{5.5} = 16.36$. She must sell 17 T-shirts to break even.

(C) How many T-shirts must she produce and sell to make a profit of $500?

Profit: $P(x) = R(x) - C(x)$

$= 9X - (3.5X + 90)$

$P(x) = 5.5X - 90$ $\Leftarrow$ Find X such that $P(x) = 500$

$500 = 5.5X - 90$

$590 = 5.5X$

$X = \frac{590}{5.5} = 107.27$

She must sell 108 T-shirts

9. Solve the equations

(A) $\log(x-3)+\log(x+3)=2$

$$\log[(x-3)(x+3)]=2$$
$$\log(x^2-9)=2$$
$$x^2-9=10^2$$
$$x^2-9=100$$
$$x^2-109=0$$
$$(x-\sqrt{109})\quad(x+\sqrt{109})=0$$
$$x=\pm\sqrt{109}$$

check answers:

$x=-\sqrt{109}$

$-\sqrt{109}-3<0$

So $\log(x-3)$ is not defined for $x=-\sqrt{109}$

$x=\sqrt{109}$

Both $\log(x-3)$ and $\log(x+3)$ are defined for $\sqrt{109}$.

Since $\sqrt{109}$ is in the domain.

$\boxed{\text{Solution: } \sqrt{109}}$

(B) $2^{x+1}=3^{2x-3}$

$$\ln 2^{x+1}=\ln 3^{2x-3}$$
$$(x+1)\ln(2)=(2x-3)\ln(3)$$
$$x\ln(2)+\ln(2)=2x\ln(3)-3\ln(3)$$
$$\ln(2)+3\ln(3)=x\ln(3)$$
$$x=\frac{\ln(2)+3\ln(3)}{\ln(3)}$$

10. Suppose the demand function for sugar is a linear function, $p = D(q)$, where p is the price in dollars and q is the quantity in thousands of pounds. When the price is \$1.2 per pounds the quantity demanded is 1 thousand pound and when the price is \$2.2 per pounds the quantity demanded is 500 pounds.

(A) Find an equation for $D(q)$

500 pounds corresponds to $q = 0.5$

$D(q) = mq + b$

The points $(1, 1.2)$ and $(0.5, 2.2)$ are points on the graph.

Slope: $m = \dfrac{2.2 - 1.2}{0.5 - 1} = \dfrac{1}{-0.5} = -2$

$y - 1.2 = -2(q - 1)$

$y = -2q + 2 + 1.2$

$y = -2q + 3.2 \Rightarrow \underline{D(q) = -2q + 3.2}$

(B) If the Supply function for sugar is

$$p = S(q) = 1.4q - 0.6,$$

find the equilibrium quantity and the equilibrium price.

$S(q) = D(q)$

$1.4q - 0.6 = -2q + 3.2$

$3.4q = 3.8$

$q = \dfrac{3.8}{3.4} = 1.12$

Equilibrium quantity is 1120 pounds

$S(1.12) = 1.4 \cdot (1.12) - 0.6 = 0.97$

Equilibrium price is $\underline{\$0.97}$