Assignment-5 solutions

1. Suppose the weekly amount paid for treatment in an animal clinic follows approximately a normal distribution with a mean of \$500 and a standard deviation of \$30.
(A) What is the probability that the weekly amount paid for treatment will exceed \$600.
(B) How much money need to be budget for next week to ensure that the probability that the budgeted amount will be exceed is 0.1 or less?

Solution (A) Let X be the weekly amount paid. Let $Z = \frac{X-600}{30}$ which is $N(0,1)$. We have

$$P(X > 600) = P\left(Z > \frac{600-500}{30}\right) = P(Z \leq 3.33) = 1 - P(Z \leq 3.33) \approx 0.00043.$$

(B) Let k be the budget for next week. We must find k such that $P(X > k) \leq 0.1$. We have

$$P(X > k) = P\left(Z > \frac{k-500}{30}\right) \leq 0.1.$$

Hence

$$P\left(Z \leq \frac{k-500}{30}\right) \geq 0.90.$$

$z_{0.10} = 1.282$ so we must solve $1.282 = \frac{k-500}{30}$ which gives $k \approx 538.5$. Thus, we must budget for at least 538.5 dollar.

2. The magnitudes of earthquakes recorded in a region of North America can be modeled by an exponential distribution with a mean of 2.4 as measured on the Richter scale. Find the probabilities that the next earthquake to strike this region will have the following characteristics:
(A) It will exceed 4.0 on the Richter scale.
(B) It will fall between 2.0 and 4.0 on the Richter scale.
(C) Find the probability that of the next 10 earthquakes to strike the region at least 1 will exceed 5.0 on the Richter scale.

Solution
(A) The mean is $\theta = 2.4$. Let X be the number on the Richter scale that the next earthquake will have. Then

$$P(X > 4) = \frac{1}{2.4}\int_4^\infty e^{-\frac{x}{2.4}}\,dx = e^{-\frac{4}{2.4}}.$$

(B)

$$P(2 < X < 4) = \frac{1}{2.4}\int_2^4 e^{-\frac{x}{2.4}}\,dx = -e^{-\frac{4}{2.4}} + e^{-\frac{2}{2.4}}.$$

(C) Let Y be the number of earthquakes to strike the region which exceed 5.0 on the Richter scale. We have

$$P(X > 5) = \frac{1}{2.4}\int_5^\infty e^{-\frac{x}{2.4}}\,dx = e^{-\frac{5}{2.4}} = 0.125.$$

The random variable Y follows a Binomial distribution with $p = 0.125$. We have

$$P(Y \geq 1) = 1 - P(Y = 0) = 1 - (1 - 0.125)^{10} = 0.74.$$

3. Wires manufactured for a certain computer system are specified to have resistance of between 0.12 and 0.14 ohm. The actual measured resistances of the wires produced by Company A have a normal probability distribution with a mean of 0.13 ohm and a standard deviation of 0.005 ohm. The actual measured resistances of the wires produced by Company B have a normal probability distribution with a mean of 0.136 ohm and a standard deviation of 0.003 ohm. The computer firm orders 70% of the wires used in its system from Company A and 30% from Company B.
(A) What is the probability that a randomly selected wire from Company B's production lot will meet the specifications?
(B) If four such wires are used in a single system and all are selected from Company B, what is the probability that all four will meet the specifications?
(C) We have that the probability that all four wires will meet the specifications when selected from company A is 0.83 (You don't have to show that). Suppose that all four wires placed in any one computer system are all from the same company and that the computer system will fail testing if any one of the four wires does not meet specifications. A computer system is selected at random and tested. It meets specifications. What is the probability that Company A's wires were used in it?
(Hint: Use Bayes's theorem).

Solution (A) Let X be the measured resistance of the wires produced by company B. Define the standard normal random variable, $Z = \frac{X-0.136}{0.003}$. The probability that a randomly selected wire from company B's production lot will meet the specification is

$$\begin{aligned} P(0.12 \le X \le 0.14) &= P\left(\frac{0.12-0.136}{0.003} \le Z \le \frac{0.14-0.136}{0.003}\right) \\ &= P(-5.33 \le Z \le 1.33) \\ &= \Phi(1.33) - \Phi(-5.33) \\ &= 0.9082. \end{aligned}$$

(B) Let Y be the number of wires used in a single system and selected from company B. The probability that all four wires will meet the specifications is,

$$P(Y=4) = (0.9082)^4 = 0.6803$$

(C) It is given that the probability that all four wires will meet the specifications when selected from company A is 0.83. Let A and B be the events that the computer firm orders the wires from company A and company B, respectively. Let G denote the event that all four wires meet the specifications. Then the probability that company A's wires were used in the randomly selected computer system given that it meets the specifications is

$$\begin{aligned} P(A \mid G) &= \frac{P(G \mid A)P(A)}{P(G)} \\ &= \frac{P(G \mid A)P(A)}{P(G \mid A)P(A) + P(G \mid B)P(B)} \\ &= \frac{(0.83)(0.70)}{(0.83)(0.70) + (0.6803)(0.30)} \\ &= 0.74. \end{aligned}$$

(Problems 2 and 3 are from the book Introduction to probability and its applications by R. Scheaffer and L. Young.)