Assignment-4

1. The random variable X of the life-lengths of a certain electronic system is associated with a probability density function of the form,

$$f(x) = \begin{cases} \frac{1}{3}e^{-\frac{x}{3}}, & x > 0; \\ 0, & \text{elsewhere.} \end{cases}$$

with measurements in 100 hours.

(A) Find the probability that the life of a particular battery of this type is less than 200 or greater than 400 hours.

Solution

$$\begin{aligned} P(X < 200 \cup X > 400) &= P(X < 200) + P(X > 400) \\ &= \int_0^2 \frac{1}{3}e^{-\frac{x}{3}}\,dx + \int_4^\infty \frac{1}{3}e^{-\frac{x}{3}} \\ &= -e^{-\frac{2}{3}} + 1 + e^{-\frac{4}{3}} \\ &= 0.75018. \end{aligned}$$

(B) Determine the Cumulative distribution function for X.

$$F(x) = \int_0^x \frac{1}{3}e^{-\frac{t}{3}}\,dt = -e^{-\frac{x}{3}} + 1$$

for $x \geq 0$.

Hence,

$$F(x) = \begin{cases} 1 - e^{-\frac{x}{3}} & \text{if } x \geq 0; \\ 0 & \text{if } x < 0. \end{cases}$$

2. Suppose the daily demand of a certain item, X, in a store sold by the pound and measured in hundreds of pounds, has a density function

$$f(x) = \begin{cases} \frac{x^2}{4}, & 0 \leq x \leq 2; \\ \frac{1}{3}, & 2 < x \leq 3; \\ 0, & \text{elsewhere} \end{cases}$$

Suppose the store's profit is \$20 for each 100 pounds sold (20 cents per pound if $X \leq 2$) and \$40 per 100 pounds if $X > 2$.

(A) Find the expected daily demand and variance of the daily demand.

(B) Find the store's expected profit for any given day.

Solution Similar to the example from class.