Homework assignment from section 1.2

Math 243,

Solution 1 (a)

If $x \leq y$, then $\min\{x, y\} = x$ and $\max\{x, y\} = y$, so $\min\{x, y\} + \max\{x, y\} = x + y$.
If $x > y$, then then $\min\{x, y\} = y$ and $\max\{x, y\} = x$, so $\min\{x, y\} + \max\{x, y\} = y + x = x + y$.
Hence $\min\{x, y\} + \max\{x, y\} = x + y$ for all $x, y \in \mathbb{R}$.

1 (b)

Let $n, m \in \mathbb{Z}$. Then we can write, $n = p_1^{n_1} p_2^{n_2} \cdots p_k^{n_k}$ and $m = p_1^{m_1} p_2^{m_2} \cdots p_k^{m_k}$ for primes p_i, with $p_1 < p_2 < \cdots < p_k$ and $n_i, m_i \in \mathbb{N}$ for $i = 1, 2, ..., k$.
(Note that if p_i is not a factor in n, then n_i=0 and similary, if p_i is not a factor in m, then m_i=0).
Then by using part 1 (a), we obtain,

$$gcd(n, m)lcm(n, m) = gcd(p_1^{n_1} \cdots p_k^{n_k}, p_1^{m_1} \cdots p_k^{m_k})lcm(p_1^{n_1} \cdots p_k^{n_k}, p_1^{m_1} \cdots p_k^{m_k})$$
$$= p_1^{\min\{n_1,m_1\}} \cdots p_k^{\min\{n_k,m_k\}} \cdot p_1^{\max\{n_1,m_1\}} \cdots p_k^{\max\{n_k,m_k\}}$$
$$= p_1^{\min\{n_1,m_1\}+\max\{n_1,m_1\}} \cdots p_k^{\min\{n_k,m_k\}+\max\{n_k,m_k\}}$$
$$= p_1^{n_1+m_1} \cdots p_k^{n_k+m_k}$$
$$= (p_1^{n_1} \cdots p_k^{n_k}) \cdot (p_1^{m_1} \cdots p_k^{m_k})$$
$$= nm$$