1. Comparison of improper integrals, chapter 7.8

0.
(a) If $\int_a^\infty f(x)\,dx = M$ and $0 \le g(x) \le f(x)$, what can you say about $\int_a^\infty g(x)\,dx$?

(b) If $\int_a^\infty g(x)\,dx = M$ and $0 \le g(x) \le f(x)$, what can you say about $\int_a^\infty f(x)\,dx$?

(c) If $\int_a^\infty f(x)\,dx = \infty$ and $0 \le g(x) \le f(x)$, what can you say about $\int_a^\infty g(x)\,dx$?

(d) If $\int_a^\infty g(x)\,dx = \infty$ and $0 \le g(x) \le f(x)$, what can you say about $\int_a^\infty f(x)\,dx$?

Determine whether the following improper integrals converge or diverge. Show all work.

1. $\int_1^\infty \frac{1}{\sqrt{x^3+4}}\,dx$

2. $\int_2^\infty \frac{2}{\sqrt{x^2-1}}\,dx$

3. $\int_3^\infty \frac{7}{(y+3)^5}\,dy$

4. $\int_0^1 \frac{1}{t^{2/3}}\,dt$

5. $\int_0^2 \frac{1}{t^2}\,dt$

6. $\int_0^1 \frac{\sin(x)+2}{x^4}\,dx$

7. $\int_2^\infty \frac{1}{\sqrt{x^2-x}}\,dx$

8. $\int_1^\infty \frac{1}{\sqrt{x^5+x^3}}\,dx$

9. $\int_0^\infty e^{-5(x+1)}\,dx$

10. $\int_2^\infty \frac{x^2-1}{x^5+2}\,dx$

11. Show that the following improper integrals converge:

(a) $\int_3^\infty e^{-\frac{x^2}{5}}\,dx$

(b) $\int_2^\infty \frac{3}{x(\ln(x))^3}\,dx$