Chapter 20

Tests of Hypotheses for One Proportion

In this chapter, we will discuss the following topic:

- Significance test for a proportion using the R-function **prop.test**.

Significance Tests for a Proportion

- To test significance for a proportion, we type

  ```
  prop.test(y,n,p)
  ```

  where y is the number of successes in the sample, n is the sample size, and p is the probability to be tested. In this test, a continuity correction is applied. Since the number of successes is an integer, using a continuous Normal distribution to approximate the sampling distribution of the estimate of p may not be entirely accurate unless n is large. A continuity correction will compensate for that.

- We can turn off the continuity correction by adding the argument **correct=FALSE** as in

  ```
  prop.test(y,n,p,correct=FALSE)
  ```

  which will be more in accordance with the textbook calculations.

Let Y be a simple random sample from a Binomial distribution with unknown success probability, p. Recall that Y is the number of successes. We can estimate the parameter p by the sample proportion,

$$\hat{p} = \frac{\text{number of successes in the sample}}{\text{sample size}} = \frac{Y}{n}.$$

The sampling distribution of $\hat{p}$ has mean p and standard deviation $\sqrt{\frac{p(1-p)}{n}}$. By the Central limit theorem, the sampling distribution of $\hat{p}$ is approximately Normal if the sample size is sufficiently large. That is $\hat{p}$ is approximately $N\left(p, \sqrt{\frac{p(1-p)}{n}}\right)$ for large n. To test the null hypothesis that the proportion p is equal to a specified value p_0, i.e., $H_0 : p = p_0$, we use the observed value z of the test statistics

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}. \tag{0.1}$$

This statistics is approximately standard normal for large values of n, when H_0 is true. The alternative hypotheses are $H_a : p > p_0$ with p-value $P(Z \geq z)$; $H_a : p < p_0$ with p-value $P(Z \leq z)$, and $H_a : p \neq p_0$ with p-value $2P(Z \geq |z|)$. A rough guide is to use the significance test for proportion when the expected number of successes, np_0, and failures, $n(1 - p_0)$, are at least 10 when H_0 hold [3].

An approximate C confidence interval for p is

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \tag{0.2}$$

where the critical value z^* is chosen such that the area under the normal density curve between $-z^*$ and z^* is C. In order to use this confidence interval, the numbers of successes and failures should both be at least 15 [3].

Problem. The Trial Urban District Assessment (TUDA) is a study sponsored by the government of student achievement in large urban school districts. The math test-score is on a scale from 0 to 500. A "basic" math level is a score of 262, a "proficient" level is a score of 299, and a "advanced" level is a score of 333. In 2011, between 1,000 and 2,700 fourth- and eighth-graders from public schools in 21 urban districts participated in the NAEP TUDA in mathematics [1].
In 2011, 792 of a random sample of 1200 eighth-graders in San Diego scored above the Basic level in mathematics. In large public city, 63% scored above the Basic level in mathematics [2]. (The study reports the proportions).

(A) Is the proportion of eighth-graders who scored above the Basic level in mathematics in San Diego significantly larger than 0.63 at the 5% level?
(B) Calculate a 99% confidence interval for the proportion of eight-graders who scored above the Basic level in mathematics in San Diego.

Solution to part (a). Let p be the proportion of eighth-graders in San Diego that scored above the Basic level in mathematics. We wish to test the null hypothesis that the proportion p is equal to 0.63 against the alternative hypothesis that p is greater than 0.63. That is, we wish to test

$$H_0 : p = 0.63 \text{ against } H_a : p > 0.63.$$

We obtain:

```
> prop.test(792,1200,0.63,alternative=c("greater"),correct=FALSE)

        1-sample proportions test without continuity correction

data:  792 out of 1200, null probability 0.63
X-squared = 4.6332, df = 1, p-value = 0.01568
alternative hypothesis: true p is greater than 0.63
95 percent confidence interval:
 0.6371695 1.0000000
sample estimates:
   p
0.66
```

Since the p-value is $0.01568 < 0.05$, we reject the null hypothesis at the 5% level. Thus, we have strong evidence that the proportion of eighth-graders who scored above the Basic level in mathematics in San Diego is greater than 0.63.

Solution to part (b). We first calculate the confidence interval using the textbook formula in (0.2): We have $0.99 + (1 - 0.99)/2 = 0.995$, so we wish to find z^* such that $P(Z \leq z^*) = 0.995$. We obtain:

```
> phat=792/1200
> z=qnorm(0.995)
> conf=c(phat-z*sqrt((phat*(1-phat))/1200),phat+z*sqrt((phat*(1-phat))/1200))
> conf
[1] 0.6247761 0.6952239
```

The 99% confidence interval for the proportion of eighth-graders who scored above the Basic level in mathematics in San Diego is (0.625, 0.695). Thus, we are 99% confident that the percent of eighth-graders who scored above the Basic level in mathematics in San Diego is between 62.5% and 69.5%.

If we do this calculation in R, we obtain:

```
> prop.test(792,1200, conf.level=0.99,correct=FALSE)

        1-sample proportions test without continuity correction

data:  792 out of 1200, null probability 0.5
X-squared = 122.88, df = 1, p-value < 2.2e-16
alternative hypothesis: true p is not equal to 0.5
99 percent confidence interval:
 0.6239822 0.6942582
sample estimates:
   p
0.66
```

R obtains a slightly different result due to the fact that R uses a different method for calculating confidence intervals for one proportion.

Explanation. The code can be explained as follows:

- The **prop.test** in R computes a chi-squared statistic, X-squared, (see chapter 22 for the definition of chi-squared statistics). This test is equivalent to the z-test when you take its square root, that is, $z = \sqrt{4.6332} = 2.152487$.
- We added the argument **correct=FALSE** to turn off the continuity correction such that the z statistics is equivalent to the textbook formula in (0.1).

References

[1] National Assessment of Educational Progress, National Center for Education Statistics, National Assessment Governing Board, Institute of Education Sciences, U.S. Department of Education at http://nationsreportcard.gov/tuda.asp

[2] National Center for Education Statistics, *The Nation's Report Card, Mathematics 2011, Trial Urban District Assessment, Results at grades 4 and 8*, Institute of Education Sciences, U.S. Department of Education, NCES 2012-452.

[3] D. S. Moore, W. I. Notz, M. A. Fligner, R. Scoot Linder. *The Basic Practice of Statistics.* W. F. Freeman and Company, New York, 2013.

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