Some notes from chapter 2.2

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1. Review of set notations

Example. Let S be the sample space, $S = \{1, 2, 3, 4, 5, 6\}$. Let A, B, C be the events $A = \{2, 4, 6\}$, $B = \{4, 5, 6\}$ and $C = \{5\}$. Then

Union: $A \cup B = \{2, 4, 5, 6\}$.
Intersection: $A \cap B = \{4, 6\}$, $A \cap C = \emptyset$ and $B \cap C = \{5\}$.
Complement: $\bar{A} = S - A$ or $\bar{A} = S \setminus A = \{1, 3, 5\}$.
$\bar{S} = \emptyset$.
$\bar{\emptyset} = S$.
$\bar{A} \cup A = S$.

We use the notation $A \cap B = AB$.
Commutative law: $A \cup B = B \cup A$ and $AB = BA$.
Associative law: $(A \cup B) \cup C = A \cup (B \cup C)$ and $(AB)C = A(BC)$.
Distributive law: $A(B \cup C) = AB \cup AC$ and $A \cup (BC) = (A \cup B) \cap (A \cup C)$.
DeMorgan's law: $\overline{A \cup B} = \bar{A} \cap \bar{B}$ or $\overline{(\cup_{i=1}^n A_i)} = \cap_{i=1}^n \bar{A}_i$,
$\overline{A \cup B} = \bar{A} \cup \bar{B}$ or $\overline{(\cap_{i=1}^n A_i)} = \cup_{i=1}^n \bar{A}_i$.
Symmetric difference: $A \Delta B = (A \setminus B) \cup (B \setminus A)$.
The symmetric difference can also be written as, $A \Delta B = (A \cup B) \setminus (A \cap B)$.
To prove this, notice that we can write, $A \setminus B = A \cap \bar{B}$
Then

$$
\begin{aligned}
&A \cup B) \setminus (A \cap B) = (A \cup B) \cap (\overline{A \cap B}) \\
&= (A \cup B) \cap (\bar{A} \cup \bar{B}) \\
&= (A \cap (\bar{A} \cup \bar{B})) \cup (B \cap (\bar{A} \cap \bar{B})) \\
&= (A \cap \bar{B}) \cup (B \cap \bar{A}),
\end{aligned}
$$

where we in the second equation used DeMorgan's law and in the third equation, the distributive law.

Example. Again, let $A = \{2, 4, 6\}$ and $B = \{4, 5, 6\}$. Then $A \Delta B = \{2, 5\}$.

Example. *Exercise* 2.13. On a large college campus, the students are able to get free copies of the school newspaper, the local newspaper, and a national paper. 82% of the students read at least one of the papers, 42% read only the school newspaper, 18% read only the local paper, 6% read only the national paper, and 1% read all three papers. Fifty-two percent read the school newspaper, and 5% read the school and national newspapers. A randomly selected student is asked whether he or she reads the school, local, or national paper. Express each of the following events in set notation and find the percentage of students represented by each.
(A) The student does not read any paper.
(B) The student reads the local paper.
(C) The student reads exactly two papers.

(D) The student reads at least two papers.

Solutions

Let S, L, N be the events that the students reads the school, local, and national papers respectively. $\bar{S}$ represents the event that the student does not read the school paper. Let P denote the probability of an event. It is given that $P(S)$=0.52. $S \cap N$ represents the event that the student reads the school and national newspaper. It is given that $P(S \cap N) = 0.05$. $S \cap L \cap N$ represents the event that the student reads all papers. It is given that $P(S \cap L \cap N) = 0.01$. $S \cup L \cup N$ represents the event that the student reads at least one newspaper. It is given that $P(S \cup L \cup N) = 0.82$.

(A) Then $\overline{S \cup L \cup N}$ represents the event that the student does not read any newspaper. We have $P(\overline{S \cup L \cup N}) = 1 - 0.82 = 0.18$. So 18% does not read any paper.

(B) Now $S \cap N \cap \bar{L}$ represents the event that the student reads the school and the national, but not the local newspaper. We have $P(S \cap N \cap \bar{L}) = 0.05 - 0.01 = 0.04$. Hence $P(L) = 1 - 0.18 - 0.42 - 0.04 - 0.06 = 0.30$. So 30% read the local paper.

(C) $P(L \cap N \cap \bar{S}) = 1 - 0.18 - 0.18 - 0.52 - 0.06 = 0.06$ and $P(L \cap S \cap \bar{N}) = 1 - 0.18 - 0.18 - 0.17 - 0.42 = 0.05$.
$(L \cap S \cap \bar{N}) \cup (S \cap N \cap \bar{L}) \cup (L \cap N \cap \bar{S})$ represents the event that the student reads exactly two papers.
We have $P((L \cap S \cap \bar{N}) \cup (S \cap N \cap \bar{L}) \cup (L \cap N \cap \bar{S})) = 0.05 + 0.04 + 0.06 = 0.15$. So 15% read exactly two papers.

(D) $(L \cap S \cap \bar{N}) \cup (S \cap N \cap \bar{L}) \cup (L \cap N \cap \bar{S}) \cup (S \cap L \cap N)$ represents the event that the student reads at least two papers.
We have $P((L \cap S \cap \bar{N}) \cup (S \cap N \cap \bar{L}) \cup (L \cap N \cap \bar{S}) \cup (S \cap L \cap N)) = 0.15 + 0.01 = 0.16$. So 16% read at least two papers.