Sample Space and Events, chapter 2.2

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Sample space

Definition

A sample space S is a set that includes all possible outcomes for a random experient listed in a mutually exclusive and exhaustive way.

Mutually exclusive: Outcomes do not overlap.
Exhaustive way: set contains all possible outcomes.

Sample space

Example

Suppose a six-sided die is rolled and the number on the upper face is observed.

$S^* = \{1, 2, 3, 4, 5, 6, even, odd\}$ is not a sample space since it is not mutually exclusive. For example 2 and even overlap.
$S^* = \{1, 2, 3, 4, 6\}$ is not a sample space since it is missing 5.

Sample spaces:
$S_1 = \{1, 2, 3, 4, 5, 6\}$
$S_2 = \{even, odd\}$

Events

We are interested in a particular outcome or a set of outcomes. These are the events of interest.

Definition

An event is any subset of a sample space.

Example

A is the event of "rolling a 6". $A = \{6\}$
B is the event of "rolling a number greater than 3". $B = \{4, 5, 6\}$
C is the event of "observing an odd number". $C = \{1, 3, 5\}$

$\emptyset$ is the empty set or the nullset. It contains no outcomes. $\emptyset$ is a subset of every set and is therefore an event.
The sample space, S, is an event in every experiment.

Events

Example

D is the event of "observing a seven" in rolling a die. $D = \emptyset$.

We say that an event, A, has occurred if the outcome of an experiment is contained in the event A.

Example

Suppose we roll a die. Let A be the event of "observing a 4".
Let B be the event of "observing a number smaller than 4".
Let C be the event of "observing an even number".
Then $A = \{4\}$.
$B = \{1, 2, 3\}$
$C = \{2, 4, 6\}$
Suppose the die shows a 4 on the upper face. Then the event A and C have occurred.

If a set has n elements, it has 2^n possible subsets.
Thus, if a sample space has n outcomes, it has 2^n possible events.

Example

Roll a dice. There are 6 outcomes and $2^6 = 64$ possible events.

Example

Toss a coin 4 times. Let A be the event of "obtaining exactly 3 heads" Let B be the event of "obtaining exactly 4 heads".
(a) Find $P(A)$ and $P(B)$
(b) Find P(at least 3 heads)
(c) Find P(fewer than 3 heads)

Example

Toss a coin 4 times. Let A be the event of "obtaining exactly 3 heads" Let B be the event of "obtaining exactly 4 heads".
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Solution

A=$\{HHHT, HHTH, HTHH, THHH\}$
B=$\{HHHH\}$
$\#S = 16$
$\#A = 4$
$\#B = 1$
$P(A) = \frac{4}{16} = \frac{1}{4}.$
$P(B) = \frac{1}{16}.$