Practice problems for Exam 2

1. Suppose the cost in dollars of producing q items is given by

$$C(q) = 1000q + 2000$$

and the price per item in dollars is given by

$$p = 6000 - 0.5q^2.$$

(A) Find the revenue function $R(q)$.

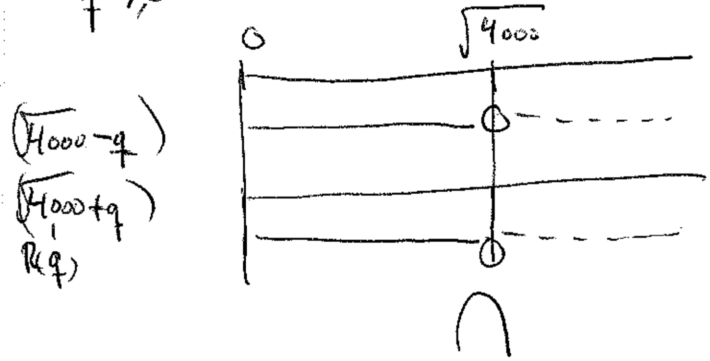
$$R(q) = p \cdot q = (6000 - 0.5q^2) \cdot q = 6000q - 0.5q^3$$

(B) Find the value of q that gives a maximum revenue.

$$R'(q) = 6000 - 3 \cdot 0.5q^2 = \frac{3}{2}(4000 - q^2) = \frac{3}{2}(\sqrt{4000} - q)(\sqrt{4000} + q) = 0$$

$$q = \pm\sqrt{4000}$$

$q \geq 0$



Since there is only one critical point ~~in the interval~~ for $q \geq 0$, there is a absolute max at $q = \underline{\sqrt{4000}}$ by the critical point theorem

Thus $q = \sqrt{4000}$ maximize revenue.

(C) Find the maximum revenue.

Max revenue:

$$R(\sqrt{4000}) = 6000 \cdot \sqrt{4000} - 0.5(\sqrt{4000})^3$$

$$\approx \underline{\$\, 252982.2}$$

(D) Find the maximum profit.

$$P(q) = R(q) - C(q) = 6000q - 0.5q^3 - 1000q - 2000$$

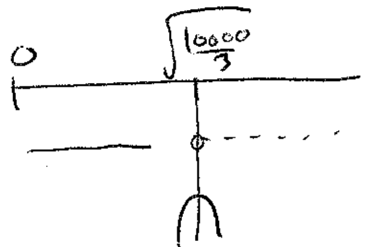
$$= 5000q - 0.5q^3 - 2000$$

$$P'(q) = 5000 - \frac{3}{2}q^2 = 0$$

$$5000 = \frac{3}{2}q^2$$

$$q^2 = \frac{10\,000}{3} \qquad q = \pm\sqrt{\frac{10000}{3}} \qquad q \geq 0$$

$P(q') >$



Test points:

$q = 0$

$P'(0) = 5000 > 0$

$q = 1000$

$P'(1000) = 5000 - \frac{3}{2}(1000)^2 = -1495000 < 0$

By the critical point theorem, there is an absolute maximum at $q = \sqrt{\frac{10000}{3}}$ since there is a relative max there and q is the only critical number for $q \geq 0$

Max profit is

$$P\left(\sqrt{\frac{10000}{3}}\right) = 5000\sqrt{\frac{10000}{3}} - 0.5\left(\sqrt{\frac{10000}{3}}\right)^3 - 2000$$

$$\approx \$\ 190450.1$$

(E) Find the marginal profit when $p = \$50$.

$P'(50)$

$$= 5000 - \frac{3}{2}(50)^2$$

$$= 1250$$

Marginal profit when $p = \$50$ is

$\$\ 1250$

2. Let $f(x) = x^2 e^{9-x^2}$.

(A) Find the relative maximum and minimum of f algebraically

$$f'(x) = 2x e^{9-x^2} - 2x e^{9-x^2} \cdot x^2$$

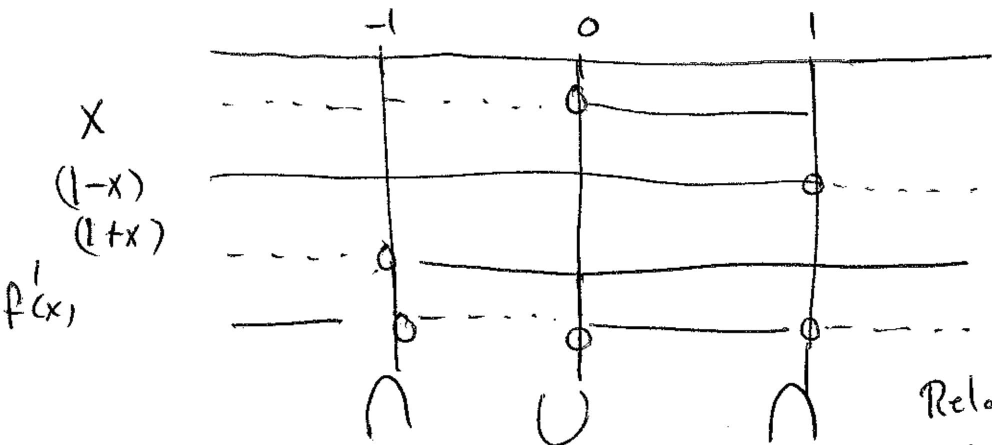
$$= 2x e^{9-x^2}(1 - x^2) = 2x e^{9-x^2}(1-x)(1+x) = 0$$

$$\Rightarrow \quad x = 0$$

$$x = \pm 1$$

Critical numbers:

$x = 0, \pm 1$



Relative max at $x = -1$

$x = 1$

Relative min at $x = 0$

Relative max are:

$f(-1) = (-1)^2 e^{9-(-1)^2} = \underline{\underline{e^8}}$

and

$f(1) = 1^2 e^{9-1^2} = \underline{\underline{e^8}}$

Relative min is

$f(0) = 0^2 e^{9-0^2} = \underline{\underline{0}}$

(B) Find where f is increasing and decreasing.

f is increasing on

$(-\infty, -1)$ and $(0, 1)$

f is decreasing on

$(-1, 0)$ and $(1, \infty)$

3. If a CD manufacturer charges $p(x)$ dollars per CD, where $p(x) = 10 - \frac{x}{4}$, then x thousand CDs will be sold.

(A) Find an expression for the total revenue from the sale of x thousand CDs.

$$R(x) = 1000 \cdot x \cdot p(x) = 1000 \cdot x \left(10 - \frac{x}{4}\right)$$

$$= 10{,}000x - 250x^2$$

(B) Find the value of x that leads to maximum revenue.

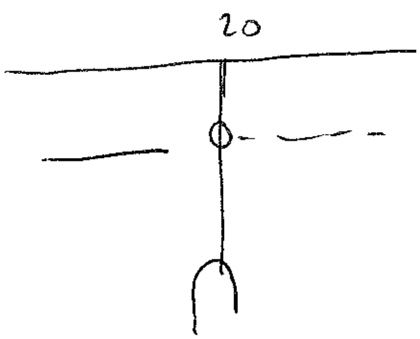
$$R'(x) = 10{,}000 - 2 \cdot 250 \cdot x = 10{,}000 - 500x$$

$$= 500(20 - x) = 0$$

$$20 - x = 0$$

$$x = 20$$

$R'(x) = 500(20-x)$



Since $x = 20$ is the only critical number ~~in the~~, ~~interval~~ and there is a relative max at $x = 20$, there must be an absolute max at $x = 20$ by the critical ~~value~~ point theorem. ~~x=20 leads to max revenue.~~

(C) Find the maximum revenue.

Max revenue is

$$R(20) = 10{,}000 \cdot 20 - 250 \cdot (20)^2$$

$$= \$\ 100000$$

Thus 20 thousand CDs leads to max revenue.

4. The total profit (in tens of dollars) from selling x books is

$$P(x) = \frac{5x-6}{2x+3}.$$

(A) Find the average profit function.

$$\overline{P}(x) = \frac{P(x)}{x} = \frac{\frac{5x-6}{2x+3}}{x} = \underline{\underline{\frac{5x-6}{(2x+3)\cdot x}}} = \frac{5x-6}{2x^2+3x}$$

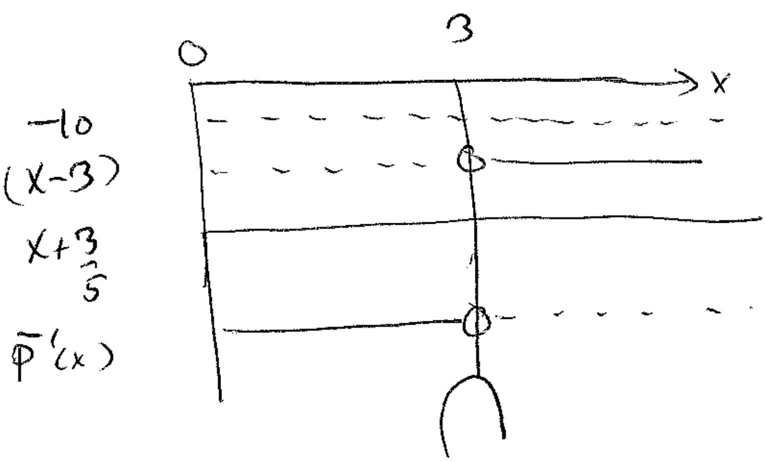
(B) Find the marginal average profit function.

$$\overline{P}'(x) = \frac{5(2x^2+3x) - (2\cdot 2x+3)(5x-6)}{(2x^2+3x)^2}$$

$$= \frac{10x^2+15x - 20x^2+24x-15x+18}{(2x^2+3x)^2} = \underline{\underline{\frac{-10x^2+24x+18}{(2x^2+3x)^2}}}$$

(C) Find the maximum average profit.

$$\overline{P}'(x) = 0 \Rightarrow \frac{-10x^2+24x+18}{(2x^2+3x)^2} = \frac{-10(x-3)(x+\frac{3}{5})}{(2x^2+3x)^2} = 0$$



$\Rightarrow \quad x-3=0 \quad$ or $\quad x+\frac{3}{5}=0$

$x=3 \quad$ or $\quad x=-\frac{3}{5}$

We have

$x \geq 0$

Relative max at $x=3$.

Since there is only one critical point for $x \geq 0$, there is a absolute max at $x=3$ by the critical point theorem.

Hence, max average profit is:

$$\overline{P}(3) = \frac{5\cdot 3-6}{2\cdot 3^2+3\cdot 3} = \frac{9}{27} = \boxed{\frac{1}{3} \text{ dollar per item}}$$

5. Find the elasticity of demand (E) for the given demand function

$$q = -\frac{\ln(\frac{p}{400})}{0.2}$$

at the indicated values of p
Determine in each case whether the demand is inelastic, elastic, or neither.

$$E = -\frac{p}{q} \cdot \frac{dq}{dp} \qquad \frac{dq}{dp} = \frac{-1}{0.2} \cdot \frac{1}{\left(\frac{p}{400}\right)} \cdot \frac{1}{400} = \frac{-1}{80} \cdot \frac{400}{p} = \frac{-5}{p}$$

Then $E = \frac{-\cancel{p}}{\frac{-\ln\left(\frac{p}{400}\right)}{0.2}} \cdot \left(\frac{-5}{\cancel{p}}\right) = \frac{-5}{\frac{\ln\left(\frac{p}{400}\right)}{0.2}} = \frac{-1}{\ln\left(\frac{p}{400}\right)}$

(A) $p = \$2$ $\quad E(2) = \frac{-1}{\ln\left(\frac{2}{400}\right)} = \underline{\underline{0.189}} < 1$

Thus demand is <u>inelastic</u>

(B) ~~$p = \$500$~~ $\quad \cancel{E(500) = \frac{-1}{\ln\left(\frac{500}{400}\right)} =}$

~~(C) $p = \$7$~~

6. A paint company has a steady annual demand for 24,500 cans of automobile primer. The comptroller for the company says that it costs \$2 to store one can of paint for 1 year and \$500 to set up the plant for the production of the primer. Find the number of cans of primer that should be produced in each batch, as well as the number of batches per year, in order to minimize total production costs.

$k=2 \quad M=24{,}500 \quad f=500$

$$q=\sqrt{\frac{2\cdot f\cdot M}{k}}=\sqrt{\frac{2\cdot(500)\cdot(24{,}500)}{2}}=3500$$

3500 cans of primer in each batch will minimize production costs.

The number of batches per year is

$$\frac{M}{q}=\frac{24{,}500}{3500}=\underline{7}$$

7. A large pharmacy has an annual need for 480 units of a certain antibiotic. It costs \$3 to store one unit for one year. The fixed cost of placing an order amounts to \$31. Find the number of units to order each time, and how many times a year the antibiotic should be ordered.

$k=3 \quad M=480 \quad f=31$

$$q=\sqrt{\frac{2f\cdot M}{k}}=\sqrt{\frac{2\cdot(31)(480)}{3}}\approx 99.6$$

$$T(q)=\frac{f\cdot M}{q}+\frac{k\cdot q}{2} \quad \text{So}$$

$$T(q)=\frac{31\cdot 480}{q}+\frac{3\cdot q}{2}=\frac{14880}{q}+\frac{3}{2}q$$

Now

$T(99)=298.803$ and $T(100)=298.800$

So $T(100)<T(99)$. So ordering 100 units each time minimizes the annual cost. The antibiotic should be ordered $\frac{M}{q}=\frac{480}{100}=4.8$ times per year or about ~~[illegible]~~ once every 2.5 months.

8. Suppose the cost in dollars of producing q items is given by

$$C(q) = 100 - 5q + \frac{1}{60}q^3.$$

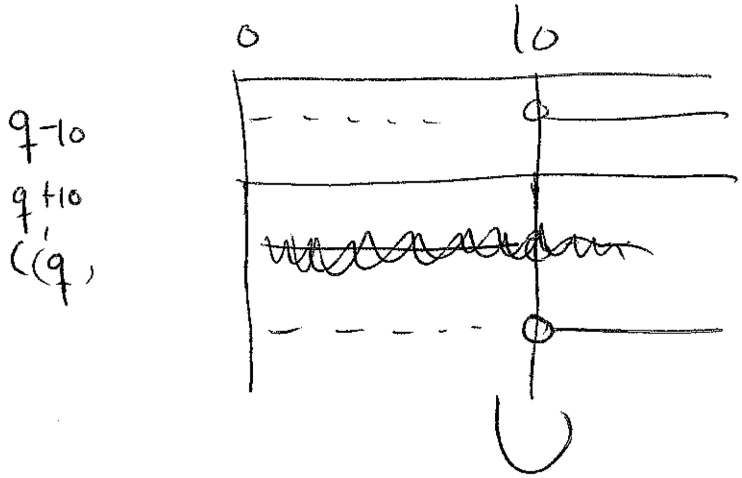
Find the minimum cost.

$$C'(q) = -5 + \frac{1}{60} \cdot 3q^2$$

$$= -5 + \frac{1}{20} q^2$$

$$= \frac{-100 + q^2}{20} = \frac{(q-10)(q+10)}{20} = 0$$

$$\Rightarrow \quad q = 10 \quad q = -10$$



Relative min at
$q = 10$.

Since q is the only critical number for $q \geq 0$, there is an absolute min at $q = 10$ by the critical point theorem

Min cost is

$$C(10) = 100 - 5 \cdot 10 + \frac{(10)^3}{60}$$

$$\approx \underline{66.7 \text{ dolla}}$$

9 Find the derivative of the following functions.

(A) $f(x) = 7x^3 - 2x + e^x + 3$

$$f'(x) = 21x^2 - 2 + e^x$$

(B) $g(x) = x \ln(x^2 + 4)$

$$g'(x) = 1 \cdot \ln(x^2+4) + \frac{1}{x^2+4} \cdot 2x \cdot x$$

$$= \ln(x^2+4) + \frac{2x^2}{x^2+4}$$

(C) $h(x) = (3x + e^{-4x})^5$

$$h'(x) = 5(3x + e^{-4x})^4 \cdot (3 - 4e^{-4x})$$

(D) $k(t) = \dfrac{te^t + 1}{t + 5}$

$$k'(t) = \frac{(1 \cdot e^t + te^t)(t+5) - 1 \cdot (te^t + 1)}{(t+5)^2}$$

$$= \frac{te^t + 5e^t + t^2e^t + 5te^t - te^t - 1}{(t+5)^2} = \frac{t^2e^t + 5te^t + 5e^t - 1}{(t+5)^2}$$