Solution to Practice for Exam 4.

1.

A. -1

B. $\frac{1}{2} + \frac{5}{2}i$

2. $P_3(x) = \sqrt{3} + \frac{1}{2}3^{-\frac{1}{2}}(x-1) - \frac{1}{4 \cdot 2!}3^{-\frac{3}{2}}(x-1)^2 + \frac{3}{8 \cdot 3!}3^{-\frac{5}{2}}(x-1)^3$

3. $\frac{1}{\sqrt{h}}(1 - \frac{x}{h} - \frac{1}{2}(\frac{x}{h})^2 - \frac{1}{2}\frac{3}{2}(\frac{x}{h})^3 - ...)$

4. $g(x) = \sum_{n=0}^{\infty} x^{2n} = 1 + x^2 + x^4 + x^6 + ...$

5. *A.* $e^{i(2\theta)} = \cos(2\theta) + i\sin(2\theta)$

B. $e^{i(2\theta)} = (e^{i\theta})^2 = \cos^2(\theta) - \sin^2(\theta) + i2\sin(\theta)\cos(\theta)$

C. $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) \quad \text{and} \quad \sin(2\theta) = 2\sin(\theta)\cos(\theta)$

6. $\frac{0.4}{1-0.4}$

7. *A.* $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$

B. $\frac{43}{72}$

C. $x^2e^x = \sum_{k=0}^{\infty} x^2\frac{x^k}{k!} = x^2 + x^3 + \frac{x^4}{2!} + \frac{x^5}{3!} + ...$

D. $(x+2)e^{x+2} = \sum_{k=0}^{\infty}(x+2)\frac{(x+2)^k}{k!} = (x+2) + (x+2)^2 + \frac{(x+2)^3}{2!} + \frac{(x+2)^4}{3!} + ...$

8.

A. $P = 0$ (unstable)

$P = 10$ (stable)

C. $P(t) = \dfrac{10e^{(0.1)t}}{9 + e^{(0.1)t}} = \dfrac{10}{9e^{-(0.1)t} + 1}$.

9.
A. $B(t) = 50000 + Ae^{(0.04)t}$, where A is an arbitrary constant.

B. $B(5) = 50000 - 20000e^{0.2}$ dollars

10. *A.* $M(t) = M_0 e^{-\frac{\ln 2}{5568}t}$

B. $t = -\dfrac{\ln 0.6}{\ln 2} 5568\text{years} \approx 4103.4$ years.