

Solution to Practice for Exam 3.

1.

A. Converges (alternating series test.)

B. Converges (ratio test)

C. Diverges (ratio test)

D. Notice, I changed this problem to $\sum_{n=0}^{\infty} \frac{\sin n}{e^n}$. Converges (comparison test and geometric series)

E. Converges (comparison test and integral test or comparison test and limit comparison test)

F. Converges (integral test)

$$2. P_3(x) = \sqrt{3} + \frac{1}{2}3^{-\frac{1}{2}}(x-1) - \frac{1}{4 \cdot 2!}3^{-\frac{3}{2}}(x-1)^2 + \frac{3}{8 \cdot 3!}3^{-\frac{5}{2}}(x-1)^3$$

$$3. \frac{1}{\sqrt{h}}\left(1 + \frac{1}{2}\frac{x}{h} + \frac{3}{8}\left(\frac{x}{h}\right)^2 + \frac{15}{48}\left(\frac{x}{h}\right)^3 + \dots\right)$$

$$4. g(x) = \sum_{n=0}^{\infty} x^{2n} = 1 + x^2 + x^4 + x^6 + \dots$$

5. A. 1

B. $2 \leq x < 4$

6. 0

$$7. A. e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

B. $\frac{43}{72}$

$$C. x^2 e^x = \sum_{k=0}^{\infty} x^2 \frac{x^k}{k!} = x^2 + x^3 + \frac{x^4}{2!} + \frac{x^5}{3!} + \frac{x^6}{4!} \dots$$

D. 30

$$E. (x+2)e^{x+2} = \sum_{k=0}^{\infty} (x+2) \frac{(x+2)^k}{k!} = (x+2) + (x+2)^2 + \frac{(x+2)^3}{2!} + \dots$$

8.

A. $P = 0$ (unstable)

$P = 10$ (stable)

$$C. P(t) = \frac{10e^t}{9+e^t} = \frac{10}{9e^{-t}+1}.$$

9.

A. $B(t) = 50000 + Ae^{(0.04)t}$, where A is an arbitrary constant.

B. $B(5) = 50000 - 20000e^{0.2}$ dollars

$$10. A. M(t) = M_0 e^{-\frac{\ln 2}{5568}t}$$

$$B. t = -\frac{\ln 0.6}{\ln 2} 5568 \text{ years} \approx 4103.4 \text{ years.}$$