1. Practice for exam 4.

1. Perform the following calculations and write your answer in the form $a + bi$.

   A.
   $(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i)^{20}$

   B.
   $\dfrac{3+2i}{i-1}$

2. Compute the Taylor polynomial of degree 3 for the function $f(x) = \sqrt{2+x}$ about 1.

3.
Use the Binomial series expansion to expand the quantity $\dfrac{1}{\sqrt{h-x}}$ about 0 in terms of the variable $\dfrac{x}{h}$. Include 4 nonzero terms.

4. Find the Taylor series about 0 for the function $g(x) = \dfrac{1}{1-x^2}$

5.
   A. Use Eulers form to rewrite $e^{i(2\theta)}$
   B. Use $e^{i(2\theta)} = (e^{i\theta})^2$ to rewrite $e^{i(2\theta)}$ in the form $a + bi$.
   C. Use your answers to parts A and B to derive two trig identities.

6. Determine the sum:
   $0.4 + (0.4)^2 + (0.4)^3 + (0.4)^4 + ... + ...$

7. *A.* Compute the Taylor series for $f(x) = e^x$ about 0.

B. Find an approximation of the integral,

$$\int_0^1 \frac{e^x - 1 - x}{x^2}\, dx,$$

by using a fourth degree ploynomial approximation for e^x about 0.

C. Use part A, to find the Taylor series for

$$g(x) = x^2 e^x$$

about 0.

D. Use the Taylor series in part C to find $g^{(6)}(0)$.

E. Use part *A*, to find the Taylor series for

$$h(x) = (x+2)e^{x+2}$$

about -2.

8. The slope field for the differential equation,

$$\frac{dP}{dt} = 0.1P(10-P)$$

is given in problem 11.2.5 in your calculus book.

A. Find the equilibrium points and determine whether they are stable or unstable.

B. Plot the solution cuves through the points
I $(0, 5)$

II $(-2, 12)$

C. Solve the differential equation with initial solution $P(0) = 1$.

9. A deposit is made to a bank account paying 4% interest compounded continuously. Suppose \$2000 is withdrawn from the account per year.

A. Set up a differential equation for the balance, B, in the account after t years.

B. Suppose the initial deposit was \$30000. How much is in the account after 5 years?

10. The half-life of the Isotope Carbon-14 is 5568 years. The Isotope Carbon-14 can be used to determine the age of organic material. Suppose the initial mass of the Isotope Carbon-14 in an old piece of wood is M_0. The rate of change of mass of the Isotope Carbon-14 with respect to time is proportional to the mass.

A. Write a differential equation for the mass, M, of the Isotope Carbon-14 present in the wood after t years.

B. Suppose the mass of the Isotope Carbon-14 in the wood is 60% of the inital amount, M_0. How old is the wood?