

1. PRACTICE FOR EXAM 3.

1. Determine if the series converges or diverges. State which convergence test you are using.

A. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n^2}$,

B. $\sum_{n=1}^{\infty} \frac{n^2}{n!}$

C. $\sum_{n=1}^{\infty} \frac{(n-2)!}{n^2}$

D. $\sum_{n=0}^{\infty} \frac{\sin n}{e^n}$

E. $\sum_{n=1}^{\infty} \frac{n^3 - 1}{n^5 + 2}$

F. $\sum_{n=2}^{\infty} \frac{3}{n(\ln n)^2}$

2. Compute the Taylor polynomial of degree 3 for the function $f(x) = \sqrt{2+x}$ about 1.

3.

Use the Binomial series expansion to expand the quantity $\frac{1}{\sqrt{h-x}}$ about 0 in terms of the variable $\frac{x}{h}$. Include 4 nonzero terms.

4. Find the Taylor series about 0 for the function $g(x) = \frac{1}{1-x^2}$

5. Given the series

$$\sum_{k=1}^{\infty} \frac{(x-3)^k}{k}$$

A. Use the ratio test to find the radius of convergence.

B Find the interval of convergence. (Make sure you check the endpoints of the interval for convergence).

6. Given the series

$$\sum_{k=1}^{\infty} \frac{(2n!)x^n}{n!n^2}$$

Use the ratio test to find the radius of convergence.

7. *A*. Compute the Taylor series for $f(x) = e^x$ about 0.

B. Find an approximation of the integral,

$$\int_0^1 \frac{e^x - 1 - x}{x^2} dx,$$

by using a fourth degree polynomial approximation for e^x about 0.

C. Use part *A*, to find the Taylor series for

$$g(x) = x^2 e^x$$

about 0.

D. Use the Taylor series in part *C* to find $g^{(6)}(0)$.

E. Use part *A*, to find the Taylor series for

$$h(x) = (x + 2)e^{x+2}$$

about -2 .

8. The slope field for the differential equation,

$$\frac{dP}{dt} = 0.1P(10 - P)$$

is given in problem 11.2.5 in your calculus book.

A. Find the equilibrium points and determine whether they are stable or unstable.

B. Plot the solution curves through the points

I $(0, 5)$

II $(-2, 12)$

C. Solve the differential equation with initial solution $P(0) = 1$.

9. A deposit is made to a bank account paying 4% interest compounded continuously. Suppose \$2000 is withdrawn from the account per year.

A. Set up a differential equation for the balance, B , in the account after t years.

B. Suppose the initial deposit was \$30000. How much is in the account after 5 years?

10. The half-life of the Isotope Carbon-14 is 5568 years. The Isotope Carbon-14 can be used to determine the age of organic material. Suppose the initial mass of the Isotope Carbon-14 in an old piece of wood is M_0 . The rate of change of mass of the Isotope Carbon-14 with respect to time is proportional to the mass.

A. Write a differential equation for the mass, M , of the Isotope Carbon-14 present in the wood after t years.

B. Suppose the mass of the Isotope Carbon-14 in the wood is 60% of the initial amount, M_0 . How old is the wood?