Calculus II

1. PRACTICE FOR EXAM 2

1. Consider the region in quandrant I, bounded by the y axis, $y = 2x^2$ and $y = 3 - x$ If the density is $\delta(x) = 1 + x$, find the total mass of the region.

2. An object travles along the curve given below. Find the total length traveled. $x = 5\cos(t^5), \qquad y = 5\sin(t^5) \qquad 0 \leq t \leq \pi.$

3. Use the comparison test to determine whether the following integrals converges of diverges. If the integral converges, find an upper bound for the integral. Show your work and explain your reasoning.

$A. \int_1^\infty \frac{\arctan x}{x^3+5}\,dx \qquad B. \int_1^\infty \frac{1}{\sqrt{y^3+y^4}}\,dy \qquad C. \int_1^\infty \frac{\sin^2 x + 1}{x^2+1}\,dx$

$D. \int_0^1 \frac{x}{\sqrt[3]{x^2+1}}\,dx \qquad E. \int_0^1 \frac{2}{y^2}\,dy \qquad F. \int_2^\infty \frac{(\sin^2 x + 1)\,x}{x^2-1}\,dx$

4. Determine if the improper integral converges or diverges. If the integral converges, evaluate the integral.

$A. \int_2^\infty \frac{dx}{x(\ln x)^2} \qquad B. \int_0^\infty \frac{1}{x^2+2}\,dx \qquad C. \int_1^\infty \frac{x}{x^2+3}\,dx$

$D. \int_1^\infty \frac{1}{\sqrt{t^2+1}}\,dt \qquad E. \int_4^\infty \frac{3+\sin y}{y}\,dy$

5. Calculate the volume of the solid generated by rotating the region in quadrant I bounded by $y = x^2$ and $y = 2 - x$ around the x-axis.

6. Calculate the volume of the solid generated by rotating the region in quadrant I bounded by $y = 2^x$, $y = 10$, and $x = 2$ around

$A.$ The line $y = 10$ $\qquad B.$ The line $y = -2$ $\qquad C.$ The line $x = 2$.

7. Calculate the volume of a cone with radius r and height h by rotating the line $y = \frac{r}{h}x$ around the x-axis.

8. Set up the integral and evaluate the integrals in the following problems:

Exercise 8.1.11

Exercise 8.1.12

Exercise 8.1.13

Exercise 8.1.14

9. The density of oil in a circular oil slick on the surface of the ocean at a distance of r meters from the center of the slick is given by $\dfrac{40}{1+r}\ \dfrac{\mathrm{kg}}{\mathrm{m}^2}$. Write an integral giving the exact mass of oil when the radius of the slick is 300 meters

10. Write a definite integral representing the area of the region bounded by $y = x$, $y = \sqrt{x}$ in quadrant I

A. with respect to x

B with respect to y

11. A conical tank with radius 2 ft and height 4 ft, pointing downward, is buried 2 feet underground. The tank contains water and the depth of the water in the tank is 1 ft. (The density of water is 62.4 Ibs/ft^3.) How much work is needed to pump the water to the surface if the variable is given as

A. The distance between the vertex of the cone and the 'slice' ?

B. The distance between the top of the cone and the 'slice' ?

C. The distance between the surface and the 'slice' ?

D. The distance between the final level of the water and the 'slice' ?

12. A cylindrical tank of radius 2 ft and height 10 ft is full of water. How much work is need to pump all of the water out of the tank to a point 1 feet above the tank? (The density of water is 62.4 Ibs/ft^3)

13. Suppose a crane, 30 ft above the ground, using chain that weighs 2 Ib/ft, lifts a 500 Ib stone to a height of 20 ft. Calculate the work required.

2. Solutions

1.
$$\int_0^1 ((3-x)-2x^2)(1+x)\,dx = \frac{11}{4}$$

2.
$5\pi^5$

3.
A. Converges, bounded above by $\frac{\pi}{2}\int_1^\infty \frac{1}{x^3}\,dx = \frac{\pi}{4}$.: Upper bound: $\frac{\pi}{4}$

B. Converges, bounded above by $\int_1^\infty \frac{1}{y^2}\,dy = 1$.: Upper bound: 1

C. Converges, bounded above by $\int_1^\infty \frac{2}{x^2}\,dx = 2$.: Upper bound: 2

D. Converges, bounded above by $\int_0^1 \frac{1}{x^{-\frac{1}{3}}}\,dx = \frac{3}{4}$.: Upper bound: $\frac{3}{4}$

E. Diverges.

F. Diverges

4.
A. Converges to $\frac{1}{\ln 2}$

B. Converges to $\frac{\sqrt{2}\pi}{4}$

C. Diverges

D. Diverges

E. Diverges

5.
$V = \pi\int_0^1((2-x)^2-(x^2)^2)\,dx = \frac{32\pi}{15}$

6.
A. $V = \pi\displaystyle\int_2^{\frac{1}{log(2)}} (10-2^x)^2\,dx$

B. $V = \pi\displaystyle\int_2^{\frac{1}{log(2)}} [(12)^2-(2^x+2)^2]\,dx$

$C.\ V = \pi \int_4^{10} \left(\frac{\log(y)}{\log(2)} - 2 \right)^2 dy$

7. $V = \pi \int_0^h (\frac{r\,x}{h})^2\, dx = \frac{\pi r^2 h}{3}$

8.
$(a)\ \pi \int_0^5 (\frac{2y}{5})^2\, dy\, cm^3 = \frac{20}{3}\pi\ cm^3$

$(b)\ \int_0^7 (20\sqrt{7^2 - y^2})\, dy\, m^3 = 245\ m^3$

$(c)\ \pi \int_0^5 (5^2 - y^2)\, dy\, mm^3 = \frac{250}{3}\pi mm^3$

$(d)\ \int_0^2 (2-y)^2\, dy\, m^3 = \frac{8}{3} m^3$

9. $\frac{(2\pi r 40)}{(1+r)}$

10.
$A.\ \int_0^1 (\sqrt{x} - x)\, dx$

$B.\ \int_0^1 (y - y^2)\, dy$
11.
$A.\ 62.4\pi \int_0^1 (\frac{1}{2}h)^2 (6-h)\, dh = \frac{7\pi \cdot 62.4}{16}$ ft-Ib

$B.\ 62.4\pi \int_3^4 (\frac{1}{2}(4-h))^2 (h+2)\, dh = \frac{7\pi \cdot 62.4}{16}$ ft-Ib

$C.\ 62.4\pi \int_5^6 (\frac{1}{2}(6-h))^2 h\, dh = \frac{7\pi \cdot 62.4}{16}$ ft-Ib

$D.\ 62.4\pi \int_0^1 (\frac{1}{2}(1-h))^2 (h+5)\, dh = \frac{7\pi \cdot 62.4}{16}$ ft-Ib

12.
$W = \int_0^{10} (2^2\pi \cdot\ 62.4\,)(11-h)\, dh = 14970\pi$ ft-Ib

13. $W = \int_0^{20} (500 + 2(30-h))\, dh = 10800$ ft-Ib