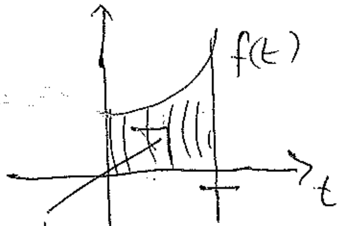

f(t) represent the rate of flow of money per unit time

Total money flow

Let t be the time, usually in years then in f(t) is in dollars per year,

8.3 Continuous Money Flow

Total Money Flow
If $f(t)$ is the rate of money flow, then the **total money flow** over the time interval from $t = 0$ to $t = T$ is given by

$$\int_0^T f(t)\,dt$$

The area under f(t) between two points in time gives the total money flow over the given time interval

The **total money flow** does not take into account the interest the money could earn after it is received.

Present Value of Money Flow
Recall that the **present value** is the amount of money that can be deposit today at a specified interest rate to yield a given sum in the future.
The present value P of an amount A compounded continuously for t years at a rate of interest r is $P = Ae^{-rt}$.

Let $f(t)$ represent the rate of the continuous flow.
Let t_i represent the time at the i^{th} subinterval with length Δt and replace A with $f(t_i)\Delta t$. Then the present value of the money flow over the i^{th} subinterval is approximately equal to $P_i = [f(t_i)\Delta t]e^{-rt_i}$. Then the total present value is approximately equal to the sum

$$\sum_{i=1}^{n} [f(t_i)\Delta t]e^{-rt_i},$$

so the present value is

$$P = \lim_{n\to\infty} \sum_{i=1}^{n} [f(t_i)\Delta t]e^{-rt_i}.$$

Present Value of Money Flow
If $f(t)$ is the rate of continuous money flow at an interest rate r for T years, then the **present value** is

$$P = \int_0^T f(t)e^{-rt}\,dt.$$

To find the **accumulated amount of money flow**, A, with interest at any time t, substitute the expression for the present value of money flow in for P in the formula $A = Pe^{rt}$ and let $t = T$.

$A = Pe^{rt}$

Accumulated Amount of Money Flow at Time T
If $f(t)$ is the rate of continuous money flow at an interest rate r at time t the **accumulated amount** of money flow at time T is

$$A = e^{rT} \int_0^T f(t)e^{-rt}\,dt.$$

The **Accumulated amount of money** A represents the accumulated value of final amount of the money flow **including** interest received on the money after it comes in.

Example. Suppose money is flowing at a constant rate of $1000 per year over 4 years at 3% interest compounded continuously, find the following.

(A) The total money flow over the 4-year period.

$\int_0^T f(t)\,dt$

$T = 4$

$f(t) = 1000$

Total money flow is $\int_0^4 1000\,dt = 1000 \cdot t \Big|_0^4$

$= 1000 \cdot 4 - 1000 \cdot 0 = 4000$

Total money flow is $\underline{\$4000}$

(B) The present value of the money flow.

$P = \int_0^T f(t)e^{-rt}\,dt$

$T = 4$

$f(t) = 1000$

$r = 0.03$

$P = \int_0^4 1000\,e^{-0.03t}\,dt$

$= 1000 \int_0^4 e^{-0.03t}\,dt = 1000 \cdot \frac{1}{-0.03} e^{-0.03t} \Big|_0^4$

$= -\frac{1000}{0.03}\left[e^{-0.03 \cdot 4} - e^{-0.03 \cdot 0}\right] \approx 3769.32$

Present value is $\underline{\$3769.32}$

(C) The accumulated amount of money flow, compounded continuously at time $T = 4$

$$A = e^{rT} \int_0^T f(t) e^{-rt}\, dt = e^{0.03 \cdot 4} \int_0^4 1000\, e^{-0.03t}\, dt$$

$T = 4$

$r = 0.03$

$f(t) = 1000$

$$= e^{0.12} \cdot 1000 \int_0^4 e^{-0.03t}\, dt$$

$$= 1000 e^{0.12} \frac{1}{-0.03} e^{-0.03t} \Big|_0^4 = \frac{-1000}{0.03} e^{0.12} \left[e^{-0.03 \cdot 4} - e^{-0.03 \cdot 0} \right)$$

$A =$ \$4249.90

with interest compounded continuously over a 4 year period.

$= 4249.90$

(D) Find the total interest earned over the 4-year period.

Total interest

= the accumulated amount − total amount of flow

$= 4249.90 - 4000 = 249.90$ dollar