

## 7.5 The Area Between Two Curves

### Area between two curves

Let  $f$  and  $g$  be continuous functions and  $f(x) \geq g(x)$  on the interval  $[a, b]$ , then the area between the curves  $f(x)$  and  $g(x)$  from  $x = a$  to  $x = b$  is given by

$$\int_a^b [f(x) - g(x)] dx.$$

**Example.** Find the area of the region bounded by  $f(x) = x^2 - 18$  and  $g(x) = x - 6$ .

### Consumers' Surplus

The market determines the price at which a product is sold. Recall that the point of intersection of the demand curve and the supply curve for a product gives the equilibrium price and the equilibrium quantity.

At the equilibrium price, the quantity that the consumers will demand of a product is the same as the quantity that the manufacturers want to sell.

Some consumers, however, would be willing to spend more for an item than the equilibrium price. The total of the differences between the higher prices that individuals would be **willing** to pay and the equilibrium price of the item, is a saving realized by those individuals and is called the **consumers' surplus**.

### Consumers' Surplus

If  $D(q)$  is a demand function with equilibrium price  $p_0$  and equilibrium demand  $q_0$ , then

$$\text{Consumers surplus} = \int_0^{q_0} [D(q) - p_0] dq.$$

If some manufacturers would be willing to supply a product a at price **lower** than the equilibrium price  $p_0$ , the total of the differences between the equilibrium price and the lower prices at which the manufacturers would sell the product is considered added income for the manufacturers and is called the **producers' surplus**.

**Producers' Surplus**

If  $S(q)$  is a supply function with equilibrium price  $p_0$  and equilibrium supply  $q_0$ , then

$$\text{Producers' surplus} = \int_0^{q_0} [p_0 - S(q)] dq.$$

**Example.** Suppose the price (in dollars per ton) for a certain product is

$$D(q) = 300 - e^{0.5q},$$

when the demand for the product is  $q$  tons. Also, suppose the function

$$S(q) = e^{0.5q} - 2$$

gives the price (in dollars per ton) when the supply is  $q$  tons. Find the consumers' surplus and the producers' surplus.

**Example.** Suppose a company wants to introduce a new machine that will produce a rate of annual savings (in dollars) given by

$$S'(t) = 50 - t^2,$$

where  $t$  is the number of years of operation of the machine, while producing a rate of annual costs (in dollars) of

$$C'(t) = t^2 + 15t$$

(A) For how many years will it be profitable to use this new machine?

(B) What are the net total saving over the entire period of use of the machine?