

7.2 Substitution

The differential of μ

If $u = f(x)$, the **differential** of u , written du is defined as

$$du = f'(x)dx.$$

Example. If $u = x^2 + 3$, then $du =$ _____

Integration by substitution

Example. Find $\int (x^2 + 3)^4 2x dx$

Example. Find $\int 8x^3 (2x^4 - 1)^5 dx$

Example. Find $\int x^2 \sqrt{x^3 - 5} dx$

How to choose u

1. u should equal some expression in the integral that, when replaced with u , tends to make the integral simpler.
2. u must be an expression whose derivative, disregarding any constant multiplier, such as the 4 in $4x^3$, is also present in the integral.

Example. Find $\int \frac{x+6}{(x^2+12x)^4} dx$

Recall that

$$\frac{d}{dx} \ln |f(x)| = \frac{f'(x)}{f(x)}.$$

The integral

$$\int \frac{f'(x)}{f(x)} dx$$

can be solved by letting u equal the expression in the denominator, $u = f(x)$, since $du = f'(x)dx$ is present in the numerator.

Example. Find $\int \frac{3x^2 + 2}{(x^3 + 2x)} dx$

Recall that

$$\frac{d}{dx}e^{f(x)} = e^{f(x)} \cdot f'(x).$$

To integrate

$$\int e^{f(x)} f'(x) dx$$

let $u = f(x)$.

Example. Find $\int x^3 e^{x^4} dx$

Substitution

Each of the following forms can be integrated using the substitution $u = f(x)$.

1. $\int [f(x)]^n f'(x) dx, n \neq -1$

Result: $\int u^n du = \frac{u^{n+1}}{n+1} + C = \frac{[f(x)]^{n+1}}{n+1} + C.$

2. $\int \frac{f'(x)}{f(x)} dx.$

Result: $\int \frac{1}{u} du = \ln |u| + C = \ln |f(x)| + C$

3. $\int e^{f(x)} f'(x) dx.$

Result: $\int e^u du = e^u + C = e^{f(x)} + C.$

Example. Find $\int x\sqrt{3-x} dx$

Substitution Method

In general, for the types of problems we are concerned with, there are three cases. We choose u to be one of the following:

1. the quantity under a root or raised to a power;
2. the quantity in the denominator;
3. the exponent on e .

Remember that some integrands may need to be rearranged to fit one of these cases.

Example. A company incurs debt at a rate of

$$D'(t) = 80(t + 5)\sqrt{t^2 + 10t}$$

dollars per year, where t is the amount of time (in years) since the company began. By the third year the company had accumulated \$12,000 in debt.

Find the total debt function.