## 7.1 Antiderivatives

The reverse of finding a derivative is known as **antidifferentiation**. The antiderivative is defined as follows:

**Antiderivative:**

If $F'(x) = f(x)$, then $F(x)$ is an **antiderivative** of $f(x)$.

**Example.** If $F(x) = x^2$, then $F'(x) = 2x$, making $F(x) = x^2$ an antiderivative of $f(x) = 2x$.

**Example.** Find an antiderivative of $f(x) = 4x^3$.

**Example.** Find three antiderivatives of $f(x) = 2x$.

If $F(x)$ and $G(x)$ are both antiderivatives of a function $f(x)$ on an interval, then there is a constant $C$ such that

$$F(x) - G(x) = C.$$

**Indefinite Integral**

If $F'(x) = f(x)$, then

$$\int f(x)\,dx = F(x) + C$$

for any real number $C$.

The symbol $\int$ is the **integral sign**

$f(x)$ is the **integrand**

$\int f(x)\,dx$ is called an **indefinite integral**

The $dx$ in the indefinite integral indicates that $\int f(x)\,dx$ is the integral of $f(x)$ with respect to the variable $x$.

**Example.** Find $\int 2ax\,dx =$ _____

**Example.** Find $\int 2ax\,da =$ _____

**Power Rule:**
For any real number $n \neq -1$,

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C.$$

**Example.** Find each indefinite integral below:

(A) $\displaystyle\int t^4 \, dt$

(B) $\displaystyle\int \frac{1}{t^3} \, dt$

(C) $\displaystyle\int \sqrt{x} \, dx$

(D) $\displaystyle\int dx$

**Constant Multiple Rule and Sum or Difference Rule:**

If all indicated integral exist,

$$\int k \cdot f(x)\,dx = k \int f(x)\,dx \quad \text{for any real number} \quad k$$

and

$$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx.$$

**Example.** Find each integral

(A) $\int 3x^2\,dx$

(B) $\int \frac{10}{z^4}\,dz$

(C) $\int (5t^7 + 3t - 4)\,dt$

(D) $\int \frac{t+2}{\sqrt{t}}\,dt$

**Indefinite Integrals of Exponential Functions:**

$$\int e^x \, dx = e^x + C$$

$$\int e^{kx} \, dx = \frac{e^{kx}}{k} + C, \quad k \neq 0$$

For $a > 0$, $a \neq 1$,

$$\int a^x \, dx = \frac{a^x}{\ln a} + C$$

$$\int a^{kx} \, dx = \frac{a^{kx}}{k(\ln a)} + C, \quad k \neq 0.$$

**Example.** Find each intergral

(A) $\int 5e^t \, dt$

(B) $\int e^{8t} \, dt$

(C) $\int 3^{-2x} \, dx$

**Indefinite Integral of $x^{-1}$:**

$$\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln|x| + C$$

**Example.** Find the integral

$$\int -\frac{2}{x}\,dx$$

**Example.** Find the cost function for the following marginal cost function, $C'(x) = 6x - 7$ with fixed cost \$5.

**Example.** Given the marginal revenue function, $R'(q) = 500 - 4e^{0.0005q}$.

(A) Find the revenue function, $R(q)$

(B) Find the demand function, $p(q)$.