6.3: Further business applications: Economic lot size; economic order quantity; elasticity of demand.

We will consider the following business applications:

1. Economic lot size. (A manufacturer must determine the production lot size that will result in minimum production and storage cost.)

2. Economic order quantity. (A purchaser must decide what quantity of an item to order that will result in minimum reordering and storage cost.)

3. Elasticity of demand. (How changes in price per item of a product affects the quantity of the product demanded.)

Economic lot size

Suppose that a company manufactures a constant number of units of a product per year and that the product can be manufactured in several batches of equal size throughout the year. The company has to consider the setup costs and warehouse costs. Many small batches increase set up cost. Large batches minimize setup costs but results in higher warehouse costs. We will use calculus to find the number of units that should be manufactured in each batch in order to minimize the total cost. This number is called the **economic lot size**.

Variables

q = number of units in each batch.
k = cost of storing one unit of the product for one year.
f = fixed setup cost to manufacture the product.
g = cost of manufacturing a single unit of the product.
M = total number of units produced annually.

The company has two types of cost:
1. Cost to manufacture the product.
2. Cost to store the finished product.

1.
The manufacturing cost per batch is: ______________

The number of batches per year is: ______________

The total annual manufacturing cost is : ___________________

2.
Since demand is constant, the inventory goes down linearly from q to 0 with an average of inventory of $\frac{q}{2}$ units per year.

The total storage cost is: ___________________

The total production cost = manufacturing cost + storage cost.

The **total production cost** of producing M units in batches of size q is,

$T(q) =$ ______________________

The domain of T is _________________

Find the value of q that will minimize $T(q)$.

Economic lot size

$$q = \sqrt{\frac{2fM}{k}}$$

is the economic lot size that minimizes total production cost,

$$T(q) = \frac{fM}{q} + gM + \frac{kq}{2}.$$

The number of batches produced per year is $\dfrac{M}{q}$.

Example. A manufacturer has a steady annual demand for $16,000$ cases of sugar. It costs \$8 to store one case for one year, \$32 in setup cost to produce each batch, and \$14 to produce each case.

(A) Find the number of cases per batch that should be produced to minimize total production cost.

(B) What is the minimum cost?

(C) Find the number of batches of sugar that should be manufactured annually.

Economic order quantity:

Now we will consider the situation when the company reorder an item that is used at a constant rate throughout the year.

The company using the product must decide:
1. how often to order,
2. how many units to request each time an order is placed.

In order words, the company must identify the **economic order quantity**.

Variables

$q =$ number of units to order each time.
$k =$ cost of storing one unit of the product for one year.
$f =$ fixed cost to place an order.
$M =$ total number of units needed each year.

Total cost=Storage cost+Reorder cost

The **total cost** of ordering M units per year with q units each time is,

$T(q) =$ ____________________

Economic order quantity

$$q = \sqrt{\frac{2fM}{k}}$$

is the economic order quantity that minimizes total ordering cost,

$$T(q) = \frac{fM}{q} + \frac{kq}{2}.$$

The number of orders placed annually is $\dfrac{M}{q}$.

Example. A large camera store sells $20,000$ batteries annually. It costs 15 cents to store one battery for one year, and \$12 to place a reorder.

(A) Find the number of batteries that should be ordered each time.

(B) How many times per year should the batteries be ordered?

Elasticity of Demand

Recall that the change in price per unit of a product affects the demand.

We will now analyze the measure of sensitivity of demand to changes in price.

The ratio of percent change in demand to percent change in price is:

$$\frac{\Delta q/q}{\Delta p/p},$$

where,
q=quantity demanded,
p = price per item,
Δq = change in q,
Δp = change in p.

This ratio is negative. Why?

We will rewrite the ratio:

Elasticity of Demand

Let $q = f(p)$, where q is demand at a price p. The **elasticity of demand** is

$$E = -\frac{p}{q} \cdot \frac{dq}{dp}$$

.

- Demand is inelastic if $E < 1$.
- Demand is elastic if $E > 1$.
- Demand has unit elasticity if $E = 1$.

An **inelastic** demand is one in which a change in price leads to a **small** change in quantity demanded.
Examples of products with an inelastic demand are necessities like food and fuel.

An **elastic** demand is one in which a change in price leads to a **large** change in quantity demanded.
Examples of products with an elastic demand are products that are purchased infrequently which can be postponed if price rises.

Unit elasticity means that the percentage changes in price and demand are relatively equal.

Example. Find the elasticity of demand E for the demand function

$$q = 200 - 0.5p^2$$

for $p = \$7$ per item.
Is the demand elastic, inelastic, or neither?

Elasticity is related to total revenue $R = pq$.

Revenue and Elasticity

Total revenue R is increasing if $\frac{dR}{dp} > 0$ which corresponds to $E < 1$
Total revenue R is decreasing if $\frac{dR}{dp} < 0$ which corresponds to $E > 1$
Total revenue R is optimized if $\frac{dR}{dp} = 0$ which corresponds to $E = 1$

- If the demand is inelastic, total revenue increases as price increases.
- If the demand is elastic, total revenue decreases as price increases.
- Total revenue is maximized at the price where demand has unit elasticity.

Example. Given the demand function

$$q = 200 - 0.5p^2.$$

Find the values of p (if any) at which total revenue is maximized.