## 6.1: Absolute extrema

- The largest possible value of a function is called the **absolute maximum**.
- The smallest possible value of a function is called the **absolute minimum**.
- A function can have several relative maxima or relative minima, but it never has more than one **absolute maximum** or **absolute minimum**.
- The **absolute maximum** or **absolute minimum** might occur at more than one value of $x$.

**Absolute maximum or minimum:**

Let $f$ be a function defined on some interval. Let $c$ be a number in the interval. Then $f(c)$ is the **absolute maximum** of $f$ on the interval if

$$f(x) \leq f(c)$$

for every $x$ in the interval, and $f(c)$ is the **absolute minimum** of $f$ on the interval if

$$f(x) \geq f(c)$$

for every $x$ in the interval.

A function has an **absolute extremum** (plura:**extrema**) at $c$ if it has either as absolute maximum or an absolute minimum there.

**Notice:**
Just like a relative extremum, an absolute extremum is a $y$-value.

**Example.**

**Extreme Value Theorem:**

A function $f$ that is continuous on a closed interval $[a, b]$ will have both an absolute maximum and an absolute minimum on the interval.

A continuous function on an **open** interval may or may not have an absolute maximum or minimum.

A discontinuous function on a closed interval may or may not have an absolute minimum or maximum.

**Finding Absolute Extrema:**

To find absolute extrema for a function $f$ that is continuous on a closed interval $[a, b]$:
1. Find all critical numbers for $f$ in $(a, b)$.
2. Evaluate $f$ for all critical numbers in $(a, b)$.
3. Evaluate $f$ for the endpoints $a$ and $b$ of the interval $[a, b]$.
4. The largest value found in Step 2 or 3 is the absolute maximum for $f$ on $[a, b]$, and the smallest value found is the absolute minimum for $f$ on $[a, b]$.

**Critical Point Theorem:**

Suppose a function $f$ is continuous on an interval $I$ and that $f$ has **exactly** one critical number in the interval $I$, located at $x = c$.
If $f$ has a relative maximum at $x = c$, then this relative maximum is the absolute maximum of $f$ on the interval $I$.
If $f$ has a relative minimum at $x = c$, then this relative minimum is the absolute minimum of $f$ on the interval $I$.

**Example.** Find the absolute extrema of the function if they exist, as well as all values of $t$ where they occur.

$$g(t) = 4t + \frac{16}{t^2} + 1 \quad \text{for} \quad t > 0.$$

**Example.** Find the minimum value of the average cost for the given cost function on the given interval,

$$C(x) = 81x^2 + 17x + 324 \quad \text{for} \quad 1 \leq x \leq 10.$$