5.12: Relative Extrema

In this section, we will determine the maximum and minimum points on the graph.

Relative Maximum or Minimum

Let c be a number in the domain of a function f. Then $f(c)$ is a **relative** (or **local**) **maximum** for f it there exists an open interval (a, b) containing c such that

$$f(x) \leq f(c)$$

for all x in (a, b).

Likewise, $f(c)$, is a **relative** (or **local**) **minimum** for f it there exists an open interval (a, b) containing c such that

$$f(x) \geq f(c)$$

for all x in (a, b).
A function has a **relative** (or **local**) **extremum** at c if it has either a relative maximum or a relative minimum there.
If c is an endpoint of the domain of f, we only consider x in the half-open interval that is in the domain.

Example. Draw the graph of the function $f(x) = 3x^4 + 8x^3 - 18x^2 + 1$ for $-4 \leq x \leq 2$. Identify the x-values of all points where the graph has a relative extrema:

Example.

Recall that the derivative of a function is the slope of the tangent line to the graph of the function. To find relative extrema we first identify all the critical numbers and endpoints.

If a function f has a relative extremum at c, then c is a critical number or c is an endpoint of the domain.

Note There is not a relative extremum at all critical numbers.
An example of that is $f(x) = x^3$.

First derivative test

We will now determine whether the critical numbers produce relative maxima, relative minima, or neither.

First derivative test:

Let c be a critical number for a function f. Suppose that f is continuous on (a, b) and differentiable on (a, b) except possibly at c, and that c is the only critical number for f in (a, b).

1. $f(c)$ is a **relative maximum** of f if the derivative $f'(x)$ is positive in the interval (a, c) and negative in the interval (c, b).

2. $f(c)$ is a **relative minimum** of f if the derivative $f'(x)$ is negative in the interval (a, c) and positive in the interval (c, b).

Example. Find all relative extrema for the following functions, as well as where each function is increasing and decreasing.

1. $f(x) = x^3 - 12x + 2$

2. $f(x) = xe^{4-x^2}$

3. $f(x) = 2x - 6x^{\frac{2}{3}}$

Example. Let the cost to produce q units of a certain product be given by

$$C(q) = 100 + 20qe^{-0.01q}$$

and let

$$p = 40e^{-0.01q}$$

be the price per unit when q units are demanded.

(A) Find the number q of units that produces maximum profit.

(B) Find the price p per unit that produces maximum profit.

(C) Find the maximum profit P.