5.1: Increasing and Decreasing Functions

A function is **increasing** if its graphs goes **up** from left to right.
A function is **decreasing** if its graph goes **down** from left to right.

Increasing and Decreasing Functions:

Let f be a function defined on some interval. Then for any two numbers, x_1 and x_2 in the interval, f is **increasing** on that interval if

$$f(x_1) < f(x_2) \quad \text{whenever} \quad x_1 < x_2,$$

and f is **decreasing** on the interval if

$$f(x_1) > f(x_2) \quad \text{whenever} \quad x_1 < x_2.$$

Example. Determine where the function $f(x) = -x^4 + 3x^3 - 2x^2 + 1$ is increasing and decreasing.

The slope of the tangent line is positive when the function is increasing.
The slope of the tangent line is negative when the function is decreasing.
The slope of the tangent line is zero at "peaks" and "valleys".

Test for intervals where $f(x)$ is increasing and decreasing:

Suppose a function f has a derivative at each point in an open interval, then

if $f'(x) > 0$ for each x in the interval, f is **increasing** on the interval.
if $f'(x) < 0$ for each x in the interval, f is **decreasing** on the interval.
if $f'(x) = 0$ for each x in the interval, f is **constant** on the interval.

The derivative $f'(x)$ can change signs from positive to negative or from negative to positive at points where $f'(x) = 0$ and at points where $f'(x)$ does not exists. The values of x where this occurs are called **critical numbers**.

Critical numbers:

The **critical numbers** for a function f are those numbers c in the domain of f for which $f'(c) = 0$ or $f'(c)$ does not exist.

A **critical point** is a point whose x-coordinate is the critical number c and whose y-coordinate is $f(c)$, i.e. $(c, f(c))$.

The following method will give an approach on how to find where a function is increasing and decreasing on an open interval:

Increasing and decreasing test:

1. Determine the critical numbers for f and put them on a number line. Determine also any points where f is undefined. These points determine several open intervals.

2. Determine whether $f'(x) > 0$ or $f'(x) < 0$ on each interval.

3. Use the test on the previous page to decide whether f is increasing or decreasing on the interval.

Example. For the following functions determine the intervals in which the function is increasing or decreasing. Locate all the points where the tangent line is horizontal.

1. $f(x) = x^3 + 3x^2 - 24x + 1$

2. $f(x) = \dfrac{x+2}{x-3}$

3. $g(x) = xe^{-2x^2}$

4. $f(t) = \sqrt{16 - t^2}$

Example. Suppose a company that sells TV's finds that the cost per TV decreases linearly with the number sold monthly, decreasing from \$500 when none are sold to \$250 when 500 are sold.

(A) Find the average cost function $\overline{C}(x)$, where x is the number of TV's sold monthly for $0 \leq x \leq 500$.

(B) Find the cost function, $C(x)$.

(C) Suppose the revenue function can be approximated by

$$R(x) = 0.0001x^3 - 0.62x^2 + 535x \quad \text{for} \quad 0 \leq x \leq 500.$$

Find the profit function, $P(x)$.

(D) Find the intervals on which $P(x)$ is increasing and decreasing. Interpret your answer.