## 4.5: Derivatives of Logarithmic Functions

In this section we will find derivatives of logarithmic functions.

Recall that the logarithmic functions and exponential functions are inverses of each other.

Also recall that $\frac{d}{dx}a^x = \ln(a)a^x$.

**Derivative of $\log_a x$:**

For any $a > 0$, $a \neq 1$, we have

$$\frac{d}{dx}[\log_a x] = \frac{1}{(\ln a)x}$$

When $a = e$, we have:

**Derivative of $\ln x$:**

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

**Example.** Find the derivatives of the following functions:

(A) $f(x) = \ln 5x$

(B) $g(x) = \log x$

Apply the chain rule to find

(A) $\dfrac{d}{dx} \log_a g(x)$

(B) $\dfrac{d}{dx} \ln g(x)$

**Example.** Apply the chain rule to find the derivative of each function.

(A) $f(x) = \ln(x^3 + 2)$

(B) $y = \log_3(\sqrt{4x^2 - 2x})$

Assume $g'(x)$ exists and $g(x) \neq 0$.
For any $a > 0$, $a \neq 1$, we have

$$\frac{d}{dx}[\log_a |x|] = \frac{1}{(\ln a)x}$$
$$\frac{d}{dx}[\ln |x|] = \frac{1}{x}$$
$$\frac{d}{dx}[\log_a |g(x)|] = \frac{1}{(\ln a)}\frac{g'(x)}{g(x)}$$
$$\frac{d}{dx}[\ln |g(x)|] = \frac{g'(x)}{g(x)}$$

**Example.** Find the derivative of the function

$$y = \ln |3x|$$

**Example.** Suppose the demand function for $q$ units of a certain item is

$$p = D(q) = 150 + \frac{30}{\ln q} \quad q > 1,$$

where $p$ is in dollars.

(A) Find the marginal revenue.

(B) Approximate the revenue from one more unit when 7 units are sold.