4.4: Derivatives of Exponential Functions

In this section we will find derivatives of exponential functions.

Derivative of e^x:

$$\frac{d}{dx}(e^x) = e^x$$

Derivative of a^x:

For any positive constant $a \neq 1$, we have

$$\frac{d}{dx}(a^x) = (\ln a)a^x.$$

The exponential function with base e has the simplest derivatives of all exponential functions.

Example. Let $f(x) = 3 \cdot 2^x$. Find $f'(x)$

We use the chain rule to prove the next two formulas:

Derivative of $a^{g(x)}$ and $e^{g(x)}$:

$$\frac{d}{dx}(a^{g(x)}) = (\ln a)a^{g(x)}g'(x)$$

and

$$\frac{d}{dx}e^{g(x)} = e^{g(x)}g'(x)$$

Example. Find the derivative of each of the following functions

(A) $g(x) = e^{4x}$

(B) $y = 3 \cdot 5^{x^2+1}$

(C) $g(x) = e^{\sqrt{x^3-1}}$.

Example. The sales, $S(t)$, in dollars of a certain product as a function of time, t, in years is given by the formula

$$S(t) = \frac{2000}{1 + 19e^{-0.5t}}.$$

(A) What is the initial sale? (The sales at time $t = 0$).

(B) What is happening to the sales as time goes on?

(C) Find the rate of change of sales at each time.

(D) Find the rate of change of sales after 3 years. Interpret your answer.

(E) What is happening to the rate of change of sales as time goes on?