4.2: Derivatives of products and quotients

In this section we will provide formulas for the derivatives of product and derivatives of quotients of functions.

The Product Rule:

Let $f(x) = u(x) \cdot v(x)$ and assume $u'(x)$ and $v'(x)$ both exists. Then

$$f'(x) = u'(x) \cdot v(x) + v'(x) \cdot u(x).$$

The Quotient Rule:

Let

$$f(x) = \frac{u(x)}{v(x)} \text{ and assume } u'(x) \text{ and } v'(x) \text{ both exists and } v(x) \neq 0.$$

Then

$$f'(x) = \frac{u'(x) \cdot v(x) - v'(x) \cdot u(x)}{[v(x)]^2}.$$

Example. Let $f(x) = (3x + 4)(4x^2)$. Use the product rule to find $f'(x)$

Example. Let $f(x) = (\sqrt{x} - 2)(5x^3 + x)$. Use the product rule to find $f'(x)$

Example. Let $f(x) = \frac{x^2+3x+2}{7x-1}$. Use the quotient rule to find $f'(x)$

Example. Let $f(x) = \frac{(x-1)(2x+4)}{x^2+3}$. Use the quotient rule to find $f'(x)$

The marginal average cost:

If the total cost to manufacture x items is given by $C(x)$, then the average cost per item is $\overline{C}(x) = \frac{C(x)}{x}$. The **marginal average cost** is the derivative of the average cost function, $\overline{C'}(x)$.

Thus, the **marginal average cost** is the rate of change of average cost.

Similarly, the **marginal average revenue function**, $\overline{R'}(x)$, is defined as the derivative of the average revenue function, $\overline{R}(x) = \frac{R(x)}{x}$.

The **marginal average profit function**, $\overline{P'}(x)$, is defined as the derivative of the average profit function, $\overline{P}(x) = \frac{P(x)}{x}$.

Example. Suppose the total cost (in hundreds of dollars) to produce x units of soap is

$$C(x) = \frac{4x+3}{x+5}.$$

(A) Find the average cost to produce x units

(B) Find the average cost to produce 10 units

(C) Find the marginal average cost function.

Example. Given the following table:

x	1	2
$g(x)$	3	5
$h(x)$	-1	2
$g'(x)$	-2	4
$h'(x)$	7	1

(A) Let $f(x) = 2g(x) + 3h(x)$. Find $f'(2)$

(B) Let $f(x) = \frac{g(x)}{h(x)}$. Find $f'(1)$