2.6: Applications: Growth and Decay; Mathematics of Finance

Recall the definition of an exponential function:

Definition. An **exponential function** with base a is defined as

$$P(t) = P_0 a^t,$$

where $a > 0$, $a \neq 1$ and P_0 is the quantity present at time $t = 0$

We have **exponential growth** when $a > 1$ and **exponential decay** when $0 < a < 1$.

We use this function when the growth or decay rate is given in "unit of time". For example the annual interest rate is 2% computed one time per year or the population is increasing by 4% per year.

Definition. Let P_0 be the quantity present at time $t = 0$. If the quantity is increasing (or decreasing) at a **continuous growth rate** (or decay), the function is given as

$$P(t) = P_0 e^{kt}.$$

If $k > 0$, we have **exponential growth**.
If $k < 0$, we have **exponential decay**.
The value of k is called the **continuous growth (decay) rate**.

Example. Convert the function $P(t) = 10(1.05)^t$ to the form $P(t) = P_0 e^{kt}$

Example. Convert the function $P(t) = 200e^{-1.7t}$ to the form $P(t) = P_0 a^t$.

Which investment is better? 6.0% compounded quartely or 5.9% compounded continuosly? The **effective rate** will help answer this question.
The effective rate is sometimes called **annual yield**.

We have that \$1 at 4% interest (per year) computed semiannually is $1(1.02)^2 = 1.0404$. The increase of 0.040 is 4.04% rather than 4% which is the interest compounded annually. 4% is called the **stated interest rate** and 4.04% is called the **effective interest rate**.

Effective rate:
If r is the annual stated rate of interest, the **effective rate of interest** is:

- 1. $r_E = (1 + \frac{r}{m})^m - 1$ when m is the number of compounding periods per year.
- 2. $r_E = e^r - 1$ when compounded continuously.

Example. Lisa decides to invest a \$7,000 bonus check into a savings account. One bank offers 6.0% compounded quartely. Another offers 5.9% compounded continuosly.

1. Which investment will earn the most interest in 5 years?

2. What is the effective rate in each case?

3. If Lisa chooses the bank with continuous compounding, how long will it take for her $\$7,000$ to grow to a least $\$10,000$.

4. If Lisa chooses the bank with quartely compounding, how long will it take for her $\$7,000$ to grow to a least $\$10,000$.

Present Value:
P is the amount that should be deposited today to produce A dollars in t years. The value of P is called the **present value** of A dollars.

Example. Curt must make a balloon payment of $\$15,000$ in 3 years. Find the present value of the payment if it includes annual interest of 7% compounded monthly.

Limited Growth Functions

Example. Sales of a new model of digital camera are approximated by

$$S(x) = 4000 - 3000e^{-x},$$

where x represents the number of years that the digital camera has been on the market, and $S(x)$ represents sales in thousands of dollars.

1. Find the sales in year 0.

2. When will sales reach $\$3,500.000$?

3. Find the limit on sales.