

2.4: Exponential Functions

Definition. An **exponential function** with base a is defined as

$$P(x) = P_0 a^x,$$

where $a > 0$, $a \neq 1$ and P_0 is the y-intercept.

We have **exponential growth** when $a > 1$ and **exponential decay** when $0 < a < 1$.

Example. Graph the exponential functions and find the y-intercepts and asymptotes:

1. $f(x) = 2^x$

2. $g(x) = -2^x$

3. $h(x) = -2^x + 1$

4. $k(x) = 2^{-x}$

5. $g(x) = -2^{-x}$

Example. Graph $f(x) = 3^x$ and $g(x) = 4^x$.

Example. Graph $P(t) = 3(4^t)$. What is the y-intercept?

Exponential equations (equations with a variable in the exponent):

Property. If $a > 0$, $a \neq 1$, and $a^x = a^y$, then $x = y$.

Example. Solve each equation:

1. $2^x = 8$

2. $3^x = \frac{1}{27}$

3. $4^{x+1} = 2(2^{5x})$

4. $27^{x^2+x} = 9^x$

- **Interest** is the return on an investment or the cost of borrowing money.
- The **Principal**, P , is the amount of money invested or borrowed.
- The **interest rate**, r is given as a percent per year.
- The **time**, t , is measured in years.

Compound interest formula:

If P dollars is invested at an annual interest rate, r per year, compounded m times per year for t years, the **compound amount** is

$$A = P \left(1 + \frac{r}{m} \right)^{tm} \text{ dollars.}$$

Example. (a) Martin invests 2000 dollar in a fund at 4% annual interest compounded one time per year. What is the compound amount in the fund after 10 years.

(b) Suppose now that the interest in part (a) is compounded 4 times per year. What is the compound amount in the fund after 10 years.

Suppose that an investment produces annual interest of 100%, so that $r = 1.00$. Suppose that you invest \$1 at this rate. Determine the amount after 1 year if the interest is compounded 1, 10, 100, 1000, 10000 and 100000 times per year.

Using the compound interest formula, we obtain

$$A = P \left(1 + \frac{r}{m}\right)^{t(m)} = 1 \left(1 + \frac{1}{m}\right)^{1(m)} = \left(1 + \frac{1}{m}\right)^m.$$

We obtain the following amounts in dollar:

m	1	10	100	1000	10000	100000
$\left(1 + \frac{1}{m}\right)^m$	2	2.593742	2.704814	2.716924	2.718146	2.718268

We see as m (the number of times compounded per year) gets larger and larger the value of the investment approaches 2.71 dollar.

Definition. $\left(1 + \frac{1}{m}\right)^m$ becomes closer and closer to the number $e \approx 2.718281828$ as m becomes larger and larger.

Example. Graph the exponential function $f(x) = e^x$

Continuous Compounding:

If P dollars is invested at an interest rate, r , compounded continuously for t years, the **compound amount** is

$$A = Pe^{rt} \text{ dollars.}$$

Example. Suppose Martin invest 2000 dollar in a fund at an interest rate of 4% compounded continuously. How much money would be in the fund after 10 years?

Example. Find the interest rate required for an investment of \$7000 to grow to \$9000 in 5 years if computed semiannually.