

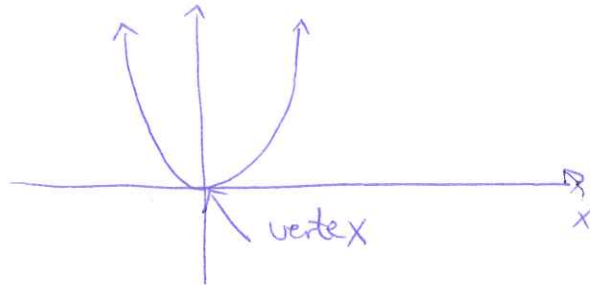
2.2 Quadratic functions; Translation and reflection

Definition. A **quadratic function** is defined by

$$f(x) = ax^2 + bx + c,$$

where a, b, c are real numbers with $a \neq 0$.

Example. Graph the quadratic function $f(x) = x^2$



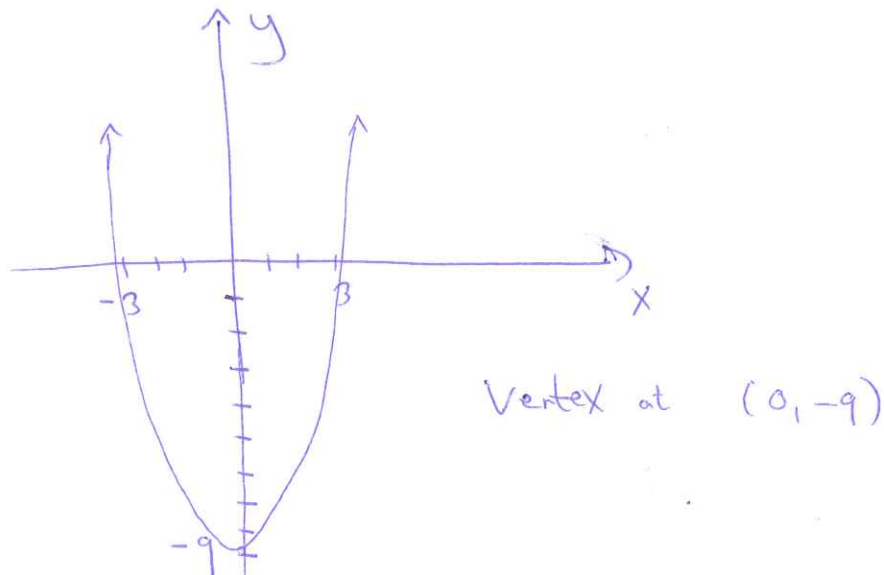
- The graph of a quadratic function is called a **parabola**.
- The maximum or minimum point on a parabola is called the **vertex** of the parabola.
- The quadratic function is symmetric with respect to a vertical line through the vertex. This line is called the **axis** of the parabola.

Example. Graph the quadratic function $f(x) = x^2 - 9$

X-intercept:

$$x^2 - 9 = 0$$

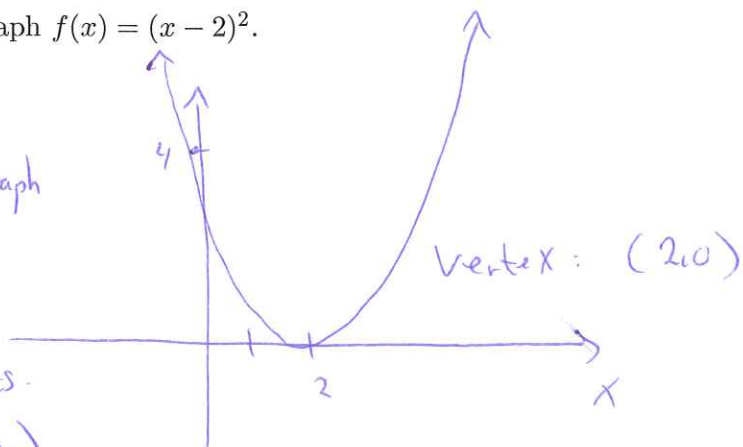
$$x = \pm 3$$



This is the graph of ~~you can see~~ $y = x^2$
translated down ¹ward by 9 units.
(vertical translation)

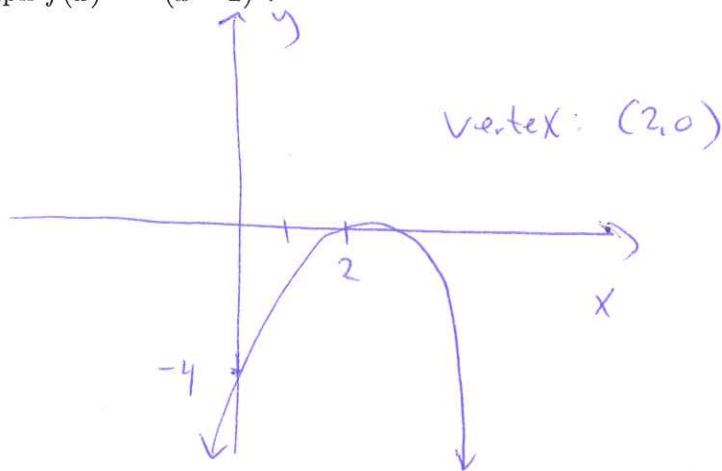
Example. Graph $f(x) = (x - 2)^2$.

This is the graph
of $y = x^2$
translated to the
right by 2 units.
(horizontal translation)



Example. Graph $f(x) = -(x - 2)^2$.

Reflection
of the graph
of $y = (x - 2)^2$
across the
X-axis.



Example. Graph $f(x) = -(x - 2)^2 + 1$.

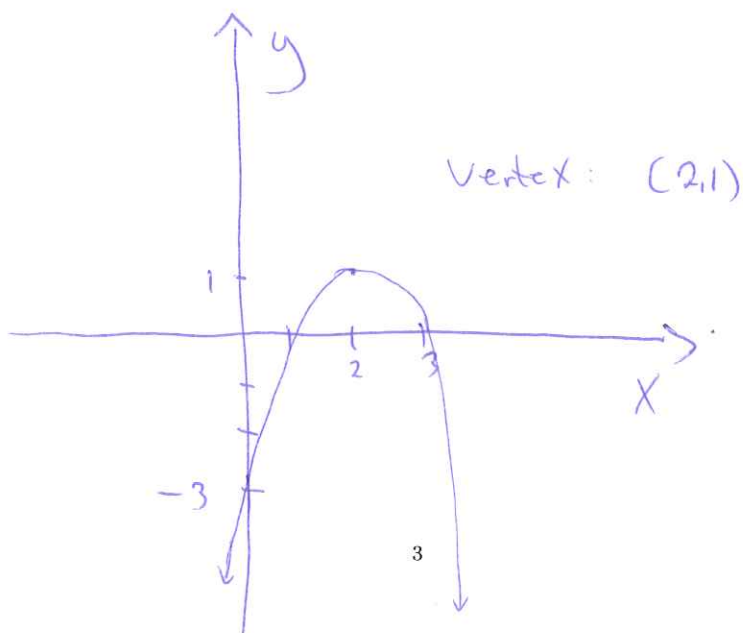
X-intercept:

$$(x - 2)^2 = 1$$

$$x - 2 = \pm 1$$

$$x = 1$$

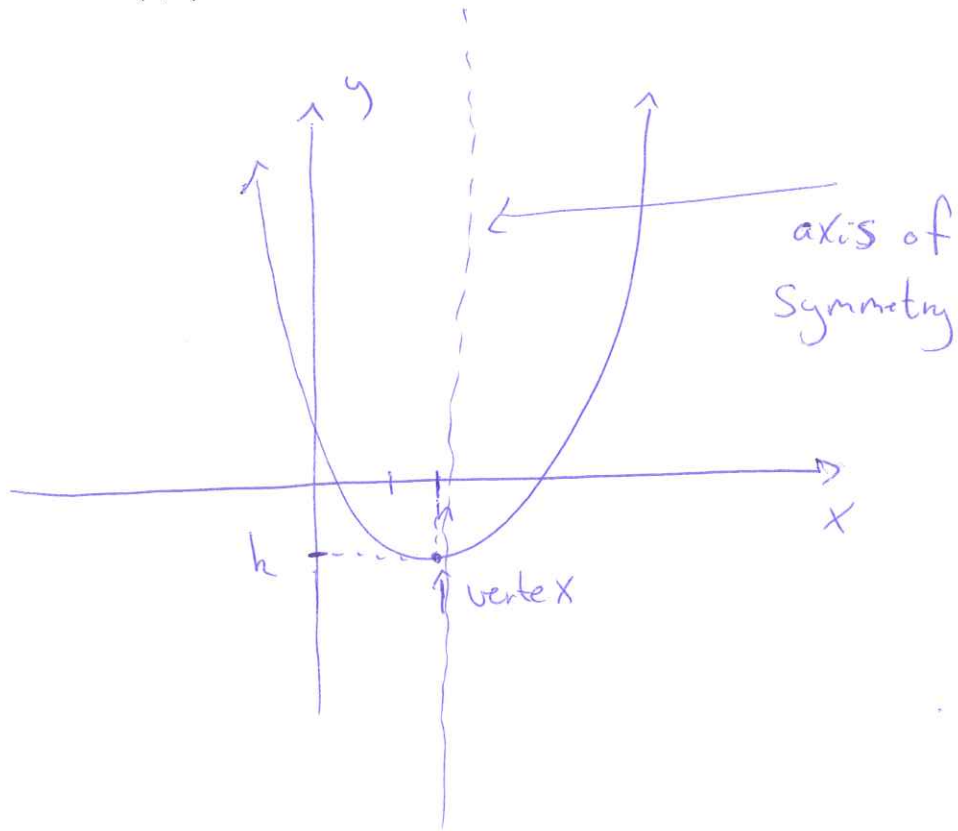
$$x = 3$$



Result 0.1. A quadratic function can be written in the form

$$f(x) = a(x - h)^2 + k,$$

where the vertex is (h, k) .



Example. Find the vertex of the function

$$f(x) = 2x^2 + 4x + 5$$

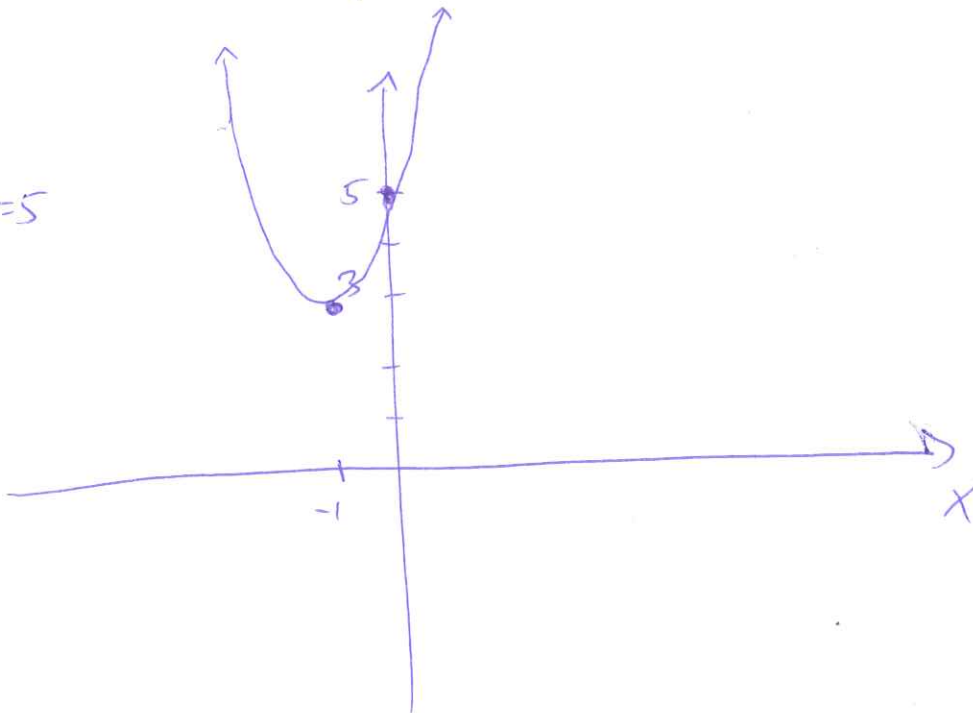
by **completing the square**. Then graph the function.

$$\begin{aligned} 2x^2 + 4x + 5 &= 2\left(x^2 + 2x + \frac{5}{2}\right) \\ &= 2\left[(x+1)^2 - 1 + \frac{5}{2}\right] \\ &= 2\left[(x+1)^2 + \frac{3}{2}\right] \\ &= 2(x+1)^2 + 3 \end{aligned}$$

Vertex: $(h, k) = (-1, 3)$

y-intercept:

$$f(0) = 2 + 3 = 5$$



Result 0.2. The graph of the quadratic function

$$f(x) = ax^2 + bx + c$$

has its vertex at

$$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right) \right)$$

The graph of f opens upward if $a > 0$ and downward if $a < 0$.

Example. A charter flight charges a fare of 300 per person plus \$2 per person for each unsold seat on the plane. The plane holds 200 passengers. Let x represent the number of unsold seats.

1. Find an expression for the total revenue for the flight $R(x)$.

$$\text{Price per person} = 300 + 2x$$

$$\text{Number of people flying: } 200 - x$$

$$R(x) = (300 + 2x)(200 - x) = 60000 + 100x - 2x^2$$

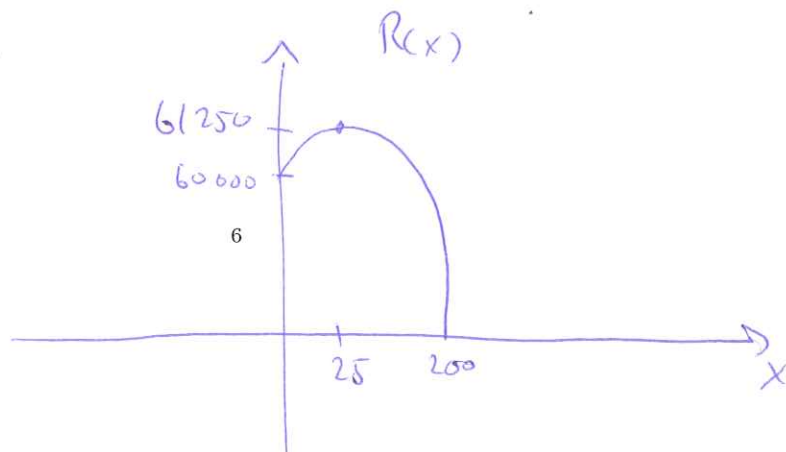
2. Find the number of unsold seats that will produce the maximum revenue.

X-coordinate of vertex: $x = \frac{-b}{2a} = \frac{-100}{-2} = 25$ seats

3. Find the maximum revenue.

$$\begin{aligned} R(25) &= 60000 + 100 \cdot 25 - 2 \cdot (25)^2 \\ &= \$61250 \quad \text{is the maximum revenue.} \end{aligned}$$

4. Draw the graph of $R(x)$.



Example. Let $C(x) = 16x + 24$ be the cost to produce x batches of widgets and let $R(x) = -4x^2 + 36x$ be the revenue in thousands of dollars.

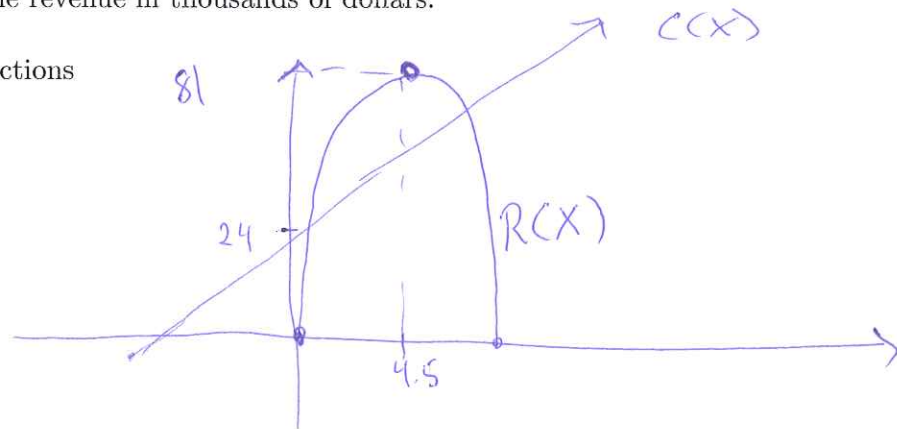
1. Graph the functions

$$R(x) = -4x^2 + 36x = 0$$

$$-4x(x-9) = 0$$

$$-4x = 0 \text{ or } x-9 = 0$$

$$x = 0 \text{ or } x = 9$$



2. Find the minimum break-even quantity

$$C(x) = R(x)$$

$$16x + 24 = -4x^2 + 36x$$

$$-4x^2 + 20x - 24 = 0$$

Divide by -4

$$x^2 - 5x + 6 = 0$$

$$(x-3)(x-2) = 0 \Rightarrow x-3 = 0 \text{ or } x-2 = 0$$

$$x = 3 \text{ or } x = 2$$

3. Find the maximum revenue

Minimum
break even
quantity,
2 batches

$$x\text{-coordinate of vertex: } \frac{-b}{2a} = \frac{-36}{2(-4)} = 4.5$$

$$R(4.5) = -4(4.5)^2 + 36(4.5)$$

$$= 81$$

Max. revenue is 81,000 dollar

4. Find the maximum profit

$$P(x) = R(x) - C(x) = -4x^2 + 36x - (16x + 24)$$

$$= -4x^2 + 20x - 24$$

$$x\text{-coordinate of vertex: } \frac{-b}{2a} = \frac{-20}{2(-4)} = 2.5$$

$$P(2.5) = -4(2.5)^2 + 20(2.5) - 24$$

Max profit is 1,000 dollar

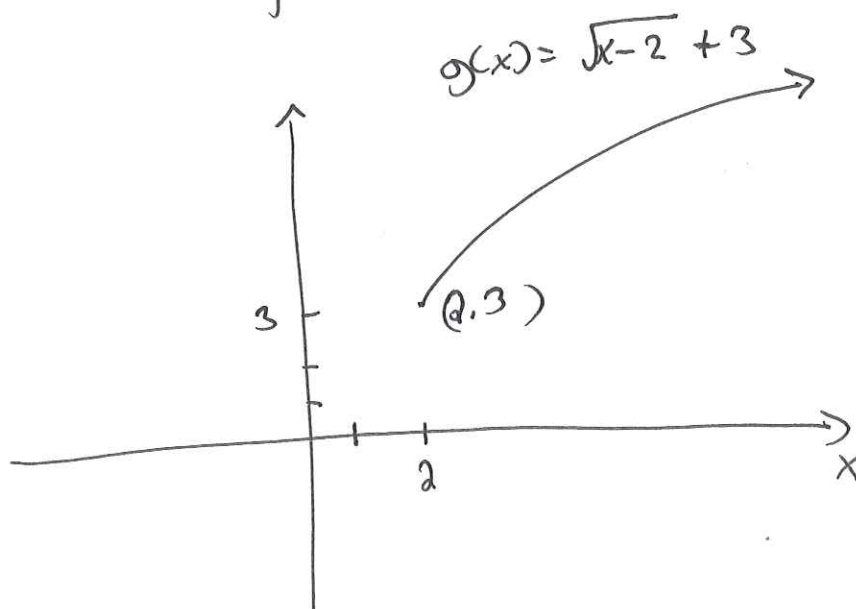
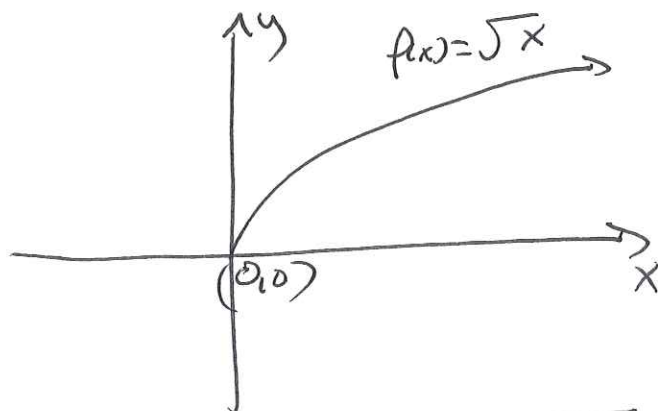
Translations and reflections:

- $f(x) + k$ Shifts the graph of $f(x)$ up k units if k is positive or down k units if k is negative.
- $f(x - h)$ Shifts the graph of $f(x)$, h units to the right if h is positive or h units to the left if h is negative.
- $-f(x)$ reflects the graph of $f(x)$ across the x-axis.
- $f(-x)$ reflects the graph of $f(x)$ across the y-axis.

Example. Draw the graph of

$$g(x) = \sqrt{x-2} + 3$$

by translating the base graph $f(x) = \sqrt{x}$.



on \sqrt{x} on $\sqrt{x-2} + 3$

$(0,0) \rightarrow (2,3)$