

2.2 Quadratic functions; Translation and reflection

Definition. A **quadratic function** is defined by

$$f(x) = ax^2 + bx + c,$$

where a, b, c are real numbers with $a \neq 0$.

Example. Graph the quadratic function $f(x) = x^2$

- The graph of a quadratic function is called a **parabola**.
- The maximum or minimum point on a parabola is called the **vertex** of the parabola.
- The quadratic function is symmetric with respect to a vertical line through the vertex. This line is called the **axis** of the parabola.

Example. Graph the quadratic function $f(x) = x^2 - 9$

Example. Graph $f(x) = (x - 2)^2$.

Example. Graph $f(x) = -(x - 2)^2$.

Example. Graph $f(x) = -(x - 2)^2 + 1$.

Result 0.1. *A quadratic function can be written in the form*

$$f(x) = a(x - h)^2 + k,$$

where the vertex is (h, k) .

Example. Find the vertex of the function

$$f(x) = 2x^2 + 4x + 5$$

by **completing the square**. Then graph the function.

Result 0.2. *The graph of the quadratic function*

$$f(x) = ax^2 + bx + c$$

has its vertex at

$$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right) \right)$$

The graph of f opens upward if $a > 0$ and downward if $a < 0$.

Example. A charter flight charges a fare of 300 per person plus \$2 per person for each unsold seat on the plane. The plane holds 200 passenger. Let x represent the number of unsold seats.

1. Find an expression for the total revenue for the flight $R(x)$.
2. Find the number of unsold seats that will produce the maximum revenue.
3. Find the maximum revenue.
4. Draw the graph of $R(x)$.

Example. Let $C(x) = 16x + 24$ be the cost to produce x batches of widgets and let $R(x) = -4x^2 + 36x$ be the revenue in thousands of dollars.

1. Graph the functions

2. Find the minimum break-even quantity

3. Find the maximum revenue

4. Find the maximum profit

Translations and reflections:

- $f(x) + k$ Shifts the graph of $f(x)$ up k units if k is positive or down k units if k is negative.
- $f(x - h)$ Shifts the graph of $f(x)$, h units to the right if h is positive or h units to the left if h is negative.
- $-f(x)$ reflects the graph of $f(x)$ across the x-axis.
- $f(-x)$ reflects the graph of $f(x)$ across the y-axis.

Example. Draw the graph of

$$g(x) = \sqrt{x - 2} + 3$$

by translating the base graph $f(x) = \sqrt{x}$.